MONTHLY NOTICES

OF THE

ROYAL ASTRONOMICAL SOCIETY.

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W. H. MAW, Esq., PRESIDENT, in the Chair.

Th. Albrecht, Königl. preuss. Geodätisches Institut, Berlin-Potsdam, Germany ;
Gustav Müller, Astrophysikalisches Observatorium, Potsdam, Germany ; and
Jean Charles Rodolphe Radau, Membre de l'Institut, 12 Rue de Tournon, Paris,

were balloted for and duly elected Associates of the Society.

Major B. F. S. Baden-Powell, 32 Princes Gate, S.W. ;
Joseph Henry Elgie, 72 Grange Avenue, Leeds ;
Frederick William Longbottom, Haslemere, Queen's Park, Leeds ;
Frederick John Marrian Stratton, B.A., Isaac Newton Student in the University of Cambridge, Caius College, Cambridge ; and
John Willis, 19 Bouverie Square, Folkestone,

were balloted for and duly elected Fellows of the Society.

The following candidates were proposed for election as Fellows of the Society, the names of the proposers from personal knowledge being appended :—

Charles Frederic Aspinwall, B.A., Chestergate, Macclesfield (proposed by E. W. Hobson) ;
Hubert Hayward Champion, Master at Uppingham School, Rutland (proposed by H. H. Turner) ;

Lieut. Alfred Henry Laurence Ferris, R.N.R., Hughenden, Coleraine Road, Westcombe Park, S.E. (proposed by P. Groves-Showell) ; and

Edward MacFarlane, Under Secretary for Lands and Chief Surveyor for New South Wales, Department of Lands, Sydney, Australia (proposed by T. F. Furber).

Sixty-eight presents were announced as having been received since the last meeting, including, amongst others :—

Publications of the West Hendon House Observatory, No. 3, presented by T. W. Backhouse ; 20 charts of the Astrographic Chart of the Heavens, presented by the Royal Observatory, Greenwich ; J. F. Pfaff, Commentatio de ortibus et occasibus siderum apud classicos commemoratis, 1786, presented by E. B. Knobel.

Note on the Annual Inequality in the Frequency of Magnetic Disturbance. By William Ellis, F.R.S.

In Mr. Maunder's paper, *Monthly Notices*, 1905 May, he suggests that the annual inequality in frequency of magnetic disturbance (maxima near the equinoxes, and minima near the solstices) "is not due to a single cause alone, but to a combination of two or more, inasmuch as the curve is not symmetrical about either the equinoxes, or the dates when the Sun's equator is on the centre of the disc." On this I would remark that in my paper, *Monthly Notices*, 1904 January, I have shown the annual inequality in frequency of magnetic disturbance to be very similar, in our latitude, to that of frequency of the aurora. Further also that, as regards the aurora, the strongly marked winter minimum of frequency in our latitude becomes less and less marked as more northern latitudes are approached, until in high latitudes the equinoctial maxima and the winter minimum become merged in one winter maximum. Remarking, then, the similarity in the seasonal frequency of magnetic disturbance to that of the aurora in our latitude it becomes a question, as I have pointed out, what happens in higher latitudes as respects frequency of magnetic disturbance. Do the equinoctial maxima and winter minimum become similarly merged, as in the case of the aurora, in one winter maximum? That is to say, as with the aurora, so also with magnetic disturbance, the effects observed at any given place may be in a way such that the latitude of the place of observation and its geographical position become to an extent dominant factors in determining the local phenomena, in addition to the relations of a more strictly cosmic character.

The Moon's observed Latitude, 1847–1901. By P. H. Cowell.

In this paper the coefficient of every term in the Moon's latitude greater than 0″·10 is obtained from the Greenwich meridian observations between 1847 and 1901. The observed motion of the node is reserved until the observations from 1750 to 1851 have been discussed. One of the largest corrections required by the present tables, however, depends upon Hansen's tabular place of the node.

Tables I., II., III. give the scheme of analysis.

Tables IV., V., VI. give the subject matter of the analysis, and Tables VII. and VIII. the results.

The errors analysed are taken directly from the Greenwich volumes and from vol. l. of the *Monthly Notices*. They are in the sense tabular minus observed for ecliptic north-polar distance, or observed minus tabular for latitude. They are subject to two important discontinuities: (i.) of tabular place when Newcomb's corrections were introduced into the *Nautical Almanac* at the beginning of 1883, the end of my period 117; (ii.) Stone's refractions were used from 1868 to 1877 inclusive.

The following references to previous papers will save much explanation:

Vol. lxiv. p. 421, the numerical values of the arguments are given.

Vol. lxv. December, a similar paper is given for the longitudes.

As an example, I follow through the third line of Table VII. It is there stated that the argument F+D or $2g-g'+2\omega-\omega'$ has a coefficient −5″·41 sin in Hansen's tables, and a coefficient −5″·36 sin in Brown's theory. In addition to this the reference number 50 is placed against the term, and two other columns are given which must be understood as meaning that when every error is multiplied by $2\sin(F+D)$ the mean is +0″·17, and when every error is multiplied by $2\cos(F+D)$ the mean is +0″·40. In order to understand the details of the arithmetic, the reference number 50 directs the reader to Table I., where it will be seen that the numerical work has been done in two independent ways for this argument, once with the help of an auxiliary angle whose movement in one period of analysis is 400×26°·311075+12°·156, or 26°·3414 in a lunar day; and a second time with the help of an auxiliary angle whose movement is 26°·1818 in a lunar day. These angles are ${}_{41}A_3$ and ${}_{55}A_4$, or angles that go through three and four revolutions in forty-one and fifty-five lunar days respectively. In Table VI. columns 41^s3, 41^c3, 55^s4, 55^c4 give the mean for each period of the errors multiplied by $2\sin {}_{41}A_3$, $2\cos {}_{41}A_3$, $2\sin {}_{55}A_4$, $2\cos {}_{55}A_4$ respectively. These mean products can be very expeditiously formed owing to the movement of the auxiliary angle in a lunar day being commensurable with 360°. In fact the average time spent

TABLE I.

Scheme of Analysis for Short-period Terms.

Ref. No.	Motion in One Lunar Day.	Excess of Argument over Auxiliary Angle. Middle of Period 44.	Movement in One Period.	Ref. No.	Motion in One Lunar Day.	Excess of Argument over Auxiliary Angle. Middle of Period 44.	Movement in One Period.
		°	°			°	°
1	79·497870	74·92	+ 6·938	22	41·759026	154·45	+ 7·959
1	79·497870	231·00	+ 34·440	23	41·079120	25·08	+ 129·761
2	77·688100	102·74	− 59·896	24	40·854141	305·91	+ 39·769
3	68·124960	301·94	+ 6·740	25	40·738880	359·60	− 6·335
3	68·124960	256·32	+ 80·172	26	39·949256	60·53	− 20·298
4	67·784720	230·84	− 55·924	26	39·949256	60·53	− 20·298
4	67·784720	143·77	− 16·545	27	39·833995	114·22	− 66·402
5	66·315190	68·08	− 140·591	27	39·833995	114·22	− 66·402
5	66·315190	177·74	+ 77·096	28	39·718734	167·91	− 112·506
6	65·974950	152·26	− 59·000	28	39·718734	262·91	+ 137·494
7	64·954804	119·39	+ 61·922	29	38·929110	111·30	− 137·446
7	64·954804	91·79	+ 135·768	29	38·929110	147·74	+ 4·076
8	64·165180	348·84	− 48·214	30	38·588870	122·26	− 132·020
8	64·165180	248·27	+ 66·072	30	38·588870	68·05	+ 6·976
9	63·145034	184·37	− 153·753	31	38·024225	192·74	+ 68·048
9	63·145034	246·43	+ 21·944	32	37·908964	246·43	+ 21·944
10	54·602040	320·70	+ 104·966	33	37·799246	113·57	− 21·944
11	54·261800	295·22	− 31·130	33	37·799246	115·49	+ 158·658
12	53·472176	53·63	+ 55·537	34	37·568724	220·95	− 114·152
13	52·792270	85·17	− 59·564	34	37·568724	222·87	+ 66·450
13	52·792270	232·37	+ 43·737	35	37·119340	62·11	− 48·816
14	52·452030	206·89	− 92·359	35	37·119340	17·20	+ 24·206
14	52·452030	2·70	+ 35·359	36	36·888818	308·11	− 13·703
15	52·111790	337·22	− 100·737	36	36·888818	346·07	+ 61·649
15	52·111790	38·51	− 24·848	37	36·779100	175·25	− 57·590
16	51·431884	94·88	+ 1·325	37	36·779100	213·21	+ 17·762
16	51·431884	69·16	+ 1·325	38	36·099194	340·07	− 169·017
17	50·642260	285·50	+ 6·904	38	36·099194	58·59	+ 39·678
17	50·642260	192·74	+ 68·048	39	35·758954	33·11	− 96·418
18	50·302020	167·26	− 68·048	40	29·365970	131·79	− 8·714
18	50·302020	189·82	− 39·192	40	29·365970	346·93	+ 70·712
19	49·622114	211·53	− 13·223	41	29·025730	321·45	− 65·384
20	48·492250	308·55	− 62·560	41	29·025730	253·54	+ 90·292
21	42·548650	46·53	+ 78·282	42	28·236106	21·42	+ 44·442
22	41·759026	19·31	− 127·560	42	28·236106	198·50	+ 73·662

Ref. No.	Motion in One Lunar Day	Excess of Argument over Auxiliary Angle. Middle of Period 44.	Movement in One Period.
		°	°
43	28·120845	75·11	− 1·662
43	28·120845	252·19	+ 27·558
44	27·556200	357·76	− 54·443
44	27·556200	89·46	+154·555
45	27·446482	224·90	− 98·330
45	27·446482	316·60	+110·668
46	27·331221	278·59	−144·434
46	27·331221	10·29	+ 64·564
47	27·215960	63·98	+ 18·459
48	26·875720	38·50	−117·637
48	26·875720	153·47	+ 83·621
49	26·426336	252·52	− 17·702
49	26·426336	326·13	+ 33·949
50	26·311075	19·82	− 12·156
50	26·311075	277·72	+ 51·704
51	26·195814	73·51	− 58·260
51	26·195814	331·41	+ 5·599
52	26·086096	198·55	− 38·288
52	26·086096	49·20	− 0·344
53	25·406190	88·05	−123·238
53	25·406190	192·74	+ 68·048
54	25·175668	300·12	− 24·161
54	25·175668	131·59	+139·233
55	25·065950	167·26	− 68·048
55	25·065950	358·73	+ 95·346
56	24·386044	20·07	−176·617
56	24·386044	20·48	+154·418
57	24·276326	247·62	+110·530
58	24·161065	301·31	+ 64·426
59	24·045804	355·00	+ 18·322
60	23·365898	63·40	− 4·291
60	23·365898	167·69	+ 56·037
61	23·256180	290·54	− 48·178
61	23·256180	34·83	+ 12·150
62	22·236034	71·34	−105·586
62	22·236034	130·12	+ 78·087
63	16·522956	247·31	−140·818
63	16·522956	136·85	+ 63·728
64	15·843050	260·71	− 15·722
64	15·843050	298·00	+ 76·351
65	15·502810	235·23	−151·818
65	15·502810	272·52	− 59·745
66	14·713186	275·83	+ 7·723
67	14·597925	329·52	− 38·381
68	14·372946	250·35	−128·373
68	14·372946	352·21	+138·788
69	14·033280	39·03	+ 2·922
70	13·693040	128·36	+ 43·254
71	13·583322	355·50	− 0·634
71	13·583322	52·98	+ 99·996
72	13·352800	102·88	− 92·842
72	13·352800	160·36	+ 7·787
73	12·788155	192·74	+ 68·048
74	12·672894	246·43	+ 21·944
75	12·563176	113·57	− 21·944
76	12·447915	167·26	− 68·048
77	12·332654	220·95	−114·152
78	11·883270	126·53	+108·147
79	11·543030	101·05	− 27·949
79	11·543030	301·37	+117·212
80	10·638145	238·25	+ 19·964
80	10·638145	23·11	+ 81·345
81	10·522884	291·94	− 26·141
81	10·522884	76·80	+ 35·241
82	9·733260	353·55	+ 1·412
82	9·733260	109·74	+153·044
83	9·502738	100·93	− 90·797
83	9·502738	217·12	+ 60·835
97	27·161150	97·05	− 3·465
98	13·747850	326·93	+ 21·924
99	13·638230	33·07	− 21·924
100	11·597841	67·98	− 6·025

TABLE II.

Scheme of Analysis for Terms of Period comparable with One Year.

Ref. No.	Motion in 40 Lunar Days. In Degrees.	In Units.	Value at Middle of Period 44. In Degrees.	In Units.
84	160°·80728	4·020182÷4	316°·68	7·9170÷4
85	120·00144	1·000012	312·54	2·6045
86	92·80520	1·031169	333·88	3·7098
87	88·41648	1−·017595	201·02	2·2336
88	79·19560	2−·020110÷2	308·40	7·7100÷2
89	65·58600	1·093100	282·92	4·7153
90	47·61064	1·058014	196·88	4·3751
91	43·00020	1−·044440	250·57	5·5682
92	38·38976	1−·040256	304·26	7·6065
93	34·00104	1·038921	171·40	5·2372

TABLE III.

Scheme of Analysis for Terms of Period comparable with Ten Years.

Ref. No.	Motion in 200 Lunar Days. In Degrees.	In Units.	Value at Middle of Period 44. In Degrees.	In Units.
94	34°·0240	1·03962	192°·74	5·8893
95	12·0804	1·00670	59·88	4·9900
96	10·9718	1·00575	246·43	22·5894
101	23·0620	1·02498	225·82	10·0364

on each number in Table VI. from the very beginning is something less than ten minutes for one computer. Lastly, to obtain the product of each error by 2 sin (F+D) we have from Table I. for the pth period

$$\sin(F+D) = \sin {}_{41}A_3 \cos\{19°·82-12°·156(p-44)\} + \cos {}_{41}A_3 \sin\{19°·82-12°·156(p-44)\}$$

In this way the result set down in the third line of Table VII. was found.

To interpret this result it is necessary to look at the two adjacent lines, the second and the fourth. The former can be at once dismissed from notice. The latter is of great importance as affecting the result. If we ignore the fourth line it would seem as if the tabular coefficient $-5''·41$ required a correction $+0''·17$, and a very large correction to its epoch. The terms $-0''·27 \sin F - 0''·72 \cos F$, however (on a principle that I have now often explained, and which lies at the root of the extremely discordant values that have been given from time to time for the

TABLE IV.

Mean Error, in Tenths of a Second of Arc, of Moon's Latitude (Observed minus Tabular) for each Column of Forty Lunar Days.

No. of Column.	1	2	3	4	5	6	7	8	9	10
850+	+ 2	+ 5	+ 9	+12	+12	+16	+ 2	− 3	− 2	+10
860	+ 3	+14	+22	+ 4	+12	− 1	− 7	− 3	+ 3	+ 5
870	− 5	+17	+18	+15	+13	+11	0	0	− 2	+12
880	+ 8	+ 5	− 9	+ 4	+10	− 4	+ 3	+ 2	+ 2	− 1
890	+ 1	− 3	− 7	− 6	− 2	+ 7	+ 5	−16	+ 2	+ 5
900	− 5	− 5	− 1	− 1	+ 7	+ 8	− 6	+ 6	− 1	+ 3
910	− 2	+ 7	+ 1	+ 7	− 2	− 2	− 5	+ 2	− 2	+ 3
920	+ 1	+ 5	−10	+ 9	0	+ 3	+ 5	− 7	− 3	+ 3
930	+ 5	+ 4	+ 3	− 5	0	+ 2	+ 3	− 2	+ 2	−10
940	0	− 3	− 1	+ 2	− 5	− 6	+ 6	− 6	0	+ 2
950	− 1	− 9	− 9	− 3	− 6	+ 3	+ 7	0	−10	+ 3
960	−12	− 3	− 4	+ 3	− 4	+10	−10	+ 5	+ 2	− 7
970	− 7	− 3	+ 1	+ 6	+ 2	+ 4	+19	+ 3	0	+ 7
980	+ 3	+10	− 1	− 2	+ 1	− 2	− 6	− 7	0	+ 1
990	+ 5	+ 1	0	0	− 7	− 7	− 6	+ 1	0	− 1
1000	0	− 3	− 1	− 5	− 8	− 2	− 5	+ 2	−10	− 1
1010	− 8	− 3	− 9	+ 1	− 9	0	− 3	+ 6	0	− 3
1020	+ 3	+ 4	+ 3	+ 5	+ 2	0	− 2	+ 4	0	+11
1030	+ 4	0	+ 7	+11	+ 9	− 2	+ 4	+15	+14	+10
1040	+13	+13	+16	+ 7	− 4	+12	+22	+ 7	+10	+15
1050	+ 9	+10	+ 9	+ 3	+ 6	+15	+12	+11	+ 7	+ 8
1060	+ 5	+14	+10	+13	+11	+17	+10	+ 2	+ 7	+ 8
1070	+ 7	+ 9	+17	+14	+18	+ 1	+10	+ 2	+ 4	− 5
1080	+ 6	+24	+30	+ 5	+ 7	+ 2	+ 1	+ 1	+14	+11
1090	+26	+20	+ 2	+22	− 7	0	− 5	+ 7	+ 7	+12
1100	+18	+11	+ 4	+ 5	− 8	+ 1	+ 1	+23	+19	+10
1110	+12	+13	+13	+ 6	+14	+24	+23	+11	+22	+ 4
1120	+ 7	+ 4	+ 3	+ 4	+10	+19	+12	− 7	+10	− 3
1130	+ 4	+ 3	[+20]	+19	+19	+ 6	− 6	− 7	−11	+ 7
1140	+ 9	+ 7	+13	+19	− 7	− 2	−27	+ 3	+ 3	+10
1150	+12	+ 2	16	−11	−20	−27	− 3	+12	+ 1	− 6
1160	−22	−16	−22	−16	− 3	+ 5	+ 8	− 6	+ 4	−11
1170	− 3	−10	− 4	− 8	− 2	−13	+ 7	− 9	− 8	−14
1180	− 3	−14	− 1	− 8	−11	− 4	− 4	+ 2	0	− 5
1190	+10	− 2	− 4	−10	+ 6	−11	− 5	+ 2	+ 1	0
1200	− 7	− 5	− 4	− 5	− 8	− 9	− 6	− 1	+ 5	+ 2
1210	− 6	−13	− 2	−16	− 2	−18	−10	− 2	0	− 8
1220	− 3	−11	− 5	− 3	− 8	− 4	+ 3	− 1	+10	+ 2
1230	− 4	− 9	− 3	− 9	− 7	−10	− 7	+ 1	− 7	+ 7
1240	0	− 4	+ 3	[0]	−14	− 2	− 2	− 7	+10	+12
1250	+ 1	− 3	− 3	− 1	−10	− 8	0	− 2	− 2	+ 5
1260	0	−11	−12	−14	− 2	− 8	− 3	+ 1	−10	+ 6
1270	−14	+ 1	−14	− 7	+ 1	+ 2	− 3	− 8	− 9	0
1280	0	+ 4	+ 6	+ 8	− 1	0	− 4	−11	+ 3	+ 5
1290	+ 9	+11	+14	+10	− 8	− 3	−10	0	+12	+ 6
1300	+10	+ 2	+ 1	− 3	− 3	− 5	+ 4	− 4	+ 8	− 3
1310	+11	− 5	− 1	− 3	+ 1	− 9	+ 8	+ 4	− 4	−12
1320	− 2	− 5	− 1	− 1	+ 6	− 4	+ 2	− 6	− 3	− 8

TABLE V.

Mean Error, in Fiftieths of a Second of Arc, of Moon's Latitude (Observed minus Tabular) for each Half-period of 200 Lunar Days.

	A.	B.	C.	D.	E.	F.
1	+40	+ 7	−28	+63	−27	−16
2	+23	− 5	0	+21	−37	− 7
3	+55	− 7	+17	+30	−37	−39
4	− 3	− 4	+13	+54	−11	−14
5	+58	−28	+31	+58	0	−33
6	+21	+ 3	+41	+84	−13	−18
7	+18	−20	+45	+28	−29	+17
8	+ 2	0	+66	+31	− 9	− 7
9	−17	− 1	+37	+65	−39	+36
10	+ 3	+33	+53	−11	−38	+ 5
11	− 5	+11	+53	+41	−30	+ 7
12	+10	−14	+44	−13	+10	0
13	+11	− 1	+65	−33	−32	+ 3
14	− 4	−13	+12	−23	−16	−13
15	+ 5	−17	+72	−79	−15	− 3
16	+ 1	−16	+29	0	+11	−19

principal elliptic coefficient), would naturally give rise to apparent terms $+0''{\cdot}13 \sin(F+D) + 0''{\cdot}35 \cos(F+D)$. The excess of the results set down in the third line, viz. $+0''{\cdot}04 \sin(F+D) + 0''{\cdot}05 \cos(F+D)$ need alone be considered. The former term makes the observed coefficient $-5''{\cdot}37$ (the agreement with Dr. Brown's coefficient being here a little closer than usual), and the cosine term may be dismissed as accidental.

In the column "concluded coefficient" I set down the observed coefficient to $0''{\cdot}1$. I consider the probable error as $\pm 0''{\cdot}03$, the actual error seldom exceeding $0''{\cdot}1$. As the theoretical coefficients have already attained a higher degree of accuracy, my Table VII. must be merely looked upon as a verification, with a determination of the principal coefficient in latitude. Table VI. may be similarly looked upon as a proof that the planetary inequalities in latitude are not large, and as a means of quickly verifying to within $0''{\cdot}1$ any inequality that may be subsequently calculated. From Table VI. I infer that there is probably no further inequality with coefficient exceeding $0''{\cdot}5$ excepting pairs with equal coefficients and arguments $A \pm F$, where A is an angle of very long period.

A few columns at the end of Table VI. require explanation.

The mean product of every error by 2 cos D has already been set down for each period in the middle of Table VI. As cos D is negative for the majority of observations, the value set down in this column will naturally follow (with opposite sign) any

TABLE VI.

Mean Value, in Tenths of a Second of Arc, for each Period of Analysis of Errors of Moon's Latitude (Observed minus Tabular), each multiplied by certain Factors.

Period.	77^s17.	77^c17.	68^s15.	68^c15.	37^s8.	37^c8.	37^s7.	37^c7.	53^s10.	53^c10.	69^s13.	69^c13.
86	+1	+2	+1	−2	−4	+2	−1	+1	+1	−1	0	+1
87	−4	+6	+6	+3	−1	−1	0	−1	−1	+1	0	−1
88	−3	−1	−2	+2	0	+2	+4	−2	+1	−3	−2	+1
89	+2	−1	+1	−3	+2	+1	0	−1	+1	−1	−1	0
90	+2	−2	+2	−3	−1	+1	+1	−1	+2	+1	+2	−1
91	0	−1	+1	−1	0	−2	−3	+1	+3	−1	+2	−2
92	−1	+4	−3	+3	+1	−3	−1	−2	−2	+2	−2	+2
93	+1	−1	+2	+1	+2	+2	0	+5	+2	+3	−2	+3
94	+5	+1	−1	+5	−2	+5	+2	+2	0	+1	0	0
95	−2	0	+1	−1	+1	+1	+2	−1	0	+1	0	0
96	+3	−1	−1	+1	+1	+1	−1	−5	+2	+5	−3	−4
97	−1	+1	+1	−1	0	−1	−1	+1	0	+1	+1	0
98	+2	−1	−2	0	−4	−2	+3	0	0	−1	+1	0
99	−1	−1	0	+1	+4	−1	+3	−1	+2	0	−1	−1
100	+1	−2	−3	0	0	0	−2	+4	−4	−4	−7	−1
101	0	−1	−2	0	−1	−1	−1	−2	0	+2	−1	+2
102	+2	0	0	−1	−1	−5	−3	+1	+1	+1	+1	0
103	−2	−2	−3	0	+5	+3	−3	+1	0	+2	−1	+1
104	+2	+1	+2	+1	+2	0	−5	−1	−2	−3	+3	0
105	−5	0	−5	−2	−4	+1	−2	−5	+6	0	−5	+3
106	−4	+3	−4	−2	+2	−2	+1	+1	0	−2	−2	+1
107	−5	+3	−3	−5	+1	−2	−6	+1	0	+5	+5	−1
108	+3	−6	+4	+5	−2	−2	−2	+3	0	+3	+3	−1
109	+6	−4	−2	+6	+4	+2	−6	0	−2	−4	−3	−3
110	+3	−1	−3	+1	+1	−3	0	+2	−3	+1	−3	+2
111	+2	+1	−1	−2	+1	+1	+1	0	−2	−1	−1	−2
112	−3	+3	+4	0	−6	−1	−1	+6	+5	−1	+3	+4
113	+1	−3	−4	−1	+3	−1	−3	−1	−3	0	+1	−2
114	−1	−5	−4	−1	−5	−2	−9	+1	−5	−7	+6	+3
115	+3	+1	+2	−2	0	+1	−1	0	0	−1	0	+1
116	0	+1	0	0	0	+1	+1	0	−1	0	0	−1
117	+1	−2	+1	−2	+1	−4	−3	−1	−1	+5	+6	0
118	+5	−1	+4	+1	−2	−2	−3	0	−2	+1	−1	+2
119	+2	+1	0	+2	+2	−1	+1	0	−1	+1	−1	+3
120	0	+1	−1	0	+2	0	−2	+2	−1	−3	+1	−3
121	−2	+1	−1	−2	−1	−6	+1	+1	−1	−1	0	−1
122	0	+4	−2	−3	+3	+2	−1	+2	+2	0	−1	+3
123	0	+2	0	−2	−6	+2	+1	−1	+2	+1	−3	0
124	+4	0	−4	−2	+1	0	−1	+1	−1	0	+1	−1
125	+1	+4	+3	−3	+3	+3	−2	−2	+2	0	−1	−2
126	+2	+1	0	−3	−1	−1	−2	+1	+2	+1	+2	−2
127	−2	+1	+1	+2	+4	+1	−1	0	−1	+1	0	+1
128	−2	−1	−3	+2	+1	0	−1	0	−2	+1	−1	+2
129	+1	−2	+1	−2	+2	−1	+1	−3	+4	−1	+5	0
130	+1	0	0	0	+1	0	+4	−1	−3	+2	−2	−3
131	−1	0	−1	0	+3	−1	+1	−1	0	0	0	0
132	−2	−2	0	−3	0	0	−1	+1	+1	+1	−1	0
133	0	0	0	0	0	−3	0	−1	+1	−2	−1	+2

Period.	27°5.	27°5.	49°9.	49°9.	50°9.	50°9.	39°7.	39°7.	28°5.	28°5.	45°8.	45°8.
86	+3	+5	−1	−7	+2	−2	−3	+2	+1	−1	0	+2
87	+4	+5	−3	+1	−3	+1	+3	+3	+2	−3	+3	+3
88	+1	0	+3	−2	−4	0	−7	+2	−7	−6	−5	+6
89	+5	−1	−2	+4	+5	−4	+6	+2	−5	−1	−5	+1
90	−1	−3	+2	−4	+3	+3	−3	+1	0	0	+1	+2
91	−5	−2	+3	+3	−3	−1	+3	+3	−3	+2	+3	+2
92	+3	0	−1	+3	−3	0	−1	+2	+2	+1	+3	0
93	+3	+3	+4	+3	+3	0	+3	−1	+3	−2	+1	+2
94	−3	−3	−3	+1	−3	+2	−1	−2	0	+2	+2	−1
95	0	+2	−4	+1	0	0	0	+1	+2	−1	+3	−3
96	−2	+1	+2	+1	+1	+1	+1	−1	+1	0	0	−1
97	+2	+2	0	0	+1	0	0	−2	+2	+1	−2	−1
98	+1	−2	0	+4	+4	+3	+2	+4	+2	+4	+3	+3
99	−2	+2	+1	+1	+2	−1	+2	+2	0	+3	0	0
100	0	0	1	+1	+1	+1	−1	0	0	0	−1	0
101	+2	0	+1	−2	+1	+2	+1	−2	0	−1	0	−1
102	−2	−3	0	0	−1	+2	0	+1	0	−2	+2	−1
103	+4	−2	0	−3	−1	0	−2	0	+1	+2	0	−1
104	0	−1	+1	−2	−1	0	0	0	+1	+2	+4	−1
105	−5	+1	−1	0	−3	+5	+1	−5	+1	+2	−1	0
106	−5	−1	0	+2	−3	−1	0	+3	0	+2	+1	−5
107	−1	0	+2	+7	−1	+4	+3	+3	0	+2	+1	0
108	0	+6	+1	+3	+1	−4	0	−3	+3	−1	+3	−2
109	0	−4	0	−3	−1	−5	+4	−2	−2	0	+2	−2
110	+3	0	+2	+2	−2	−1	+2	−1	+1	0	−1	−0
111	−1	+2	+1	0	−2	+1	+2	+3	−1	−4	−1	−2
112	−2	−1	0	+1	+2	−5	+1	−5	+1	−6	+5	+4
113	−1	+5	+3	+3	+2	−1	+4	0	−2	+5	+4	−1
114	+1	0	+1	0	0	−3	+4	−2	−1	−1	+3	−2
115	+5	+2	0	+4	+1	−4	0	+3	−2	+4	−3	−3
116	−1	+3	+5	+3	−8	−3	−2	+10	−3	−10	−2	+9
117	+2	−1	+2	−2	0	+1	+2	+1	0	−4	−3	−2
118	−5	−1	−2	−1	−2	+1	−3	+1	−2	−1	+1	−2
119	0	+4	0	+1	0	+4	−4	−2	+4	+1	−2	0
120	−3	+1	+1	−2	+1	+1	−2	−1	−2	+2	+3	+3
121	−2	+2	+1	+1	+3	0	−1	−3	+3	0	+2	+3
122	−2	−2	+1	+1	−2	+2	−2	+3	+3	+1	−2	−2
123	+2	−2	+2	−1	−1	−2	+1	−1	+2	0	+1	−2
124	+4	+2	+2	−4	+2	+1	−3	0	+3	−3	+4	+1
125	−3	+1	0	0	0	0	−1	+1	+2	+1	−3	−1
126	−1	0	+1	0	−2	−1	−1	+2	−1	+3	+2	+1
127	−2	+1	+1	+2	−1	−1	−2	−1	+3	0	+2	+1
128	+2	0	+1	+2	−2	+1	−1	−2	−1	−2	+3	+4
129	+1	+3	+2	+1	−5	0	+2	−4	+2	−1	−2	+1
130	−1	−1	+3	+1	0	+2	−1	−2	+1	+2	−1	+1
131	+2	+2	0	−1	+1	+1	+1	0	+2	−1	0	+2
132	0	+2	−3	−1	−2	+2	−1	+3	−3	−1	+1	+2
133	+1	+2	0	+2	0	−1	0	0	0	−1	−1	−2

Period.	68s12.	68s12.	sin 5D.	cos 5D.	53s8.	53c8.	27s4.	27c4.	68s10.	68c10.	41s6.	41c6.
86	+5	+3	+1	−6	+1	−7	+2	+1	0	+2	−2	−1
87	−2	+4	−3	0	+1	−2	+5	−2	0	−3	−2	+1
88	−3	+3	0	+3	0	−4	+6	−3	+2	+2	+1	0
89	+1	−5	+2	−4	−1	+2	+4	+2	−4	−2	+1	−5
90	+3	+2	−1	−3	0	−5	0	−2	+1	−3	−3	+2
91	0	−1	+3	0	−2	+2	0	−2	0	+2	+1	+3
92	+2	0	−1	−1	−1	+1	+1	+2	0	0	0	−1
93	0	+1	0	+1	−1	0	−1	0	0	−1	0	+1
94	+2	+2	+1	−3	+1	−1	−1	0	−2	−3	−4	+2
95	−4	+3	−2	0	+2	+2	−1	−4	+3	−1	+2	0
96	+4	−6	−4	+6	+1	+2	+3	+1	+1	−5	+3	+2
97	+2	−1	+2	−2	0	+3	+1	+4	0	0	−1	+2
98	+1	+1	−2	+2	0	+2	0	+1	0	+1	+1	−1
99	−3	0	−2	+2	0	−5	−3	−4	−5	−3	0	−3
00	0	−1	0	+1	−1	−3	−1	+3	−1	−2	+1	+2
01	0	−1	+2	−1	−1	−3	+1	−1	−1	−2	−2	+2
02	+2	−1	−2	+3	+4	−2	+4	+4	−3	+2	−1	0
03	−2	−2	−1	−1	+1	−2	−2	+1	+2	+2	−2	+1
04	+1	+3	−1	0	−1	+1	−2	2	0	−2	0	+2
05	−3	0	−3	−1	+3	+1	0	−6	+1	+2	+2	−2
06	+6	+5	−4	−2	−4	−1	+1	−2	+1	−3	+4	−1
07	0	+3	+1	+4	+3	0	−2	−2	−1	0	+3	+1
08	+3	+1	−3	+2	+2	+1	−1	−2	−1	+1	+4	+2
09	−4	+1	−3	+3	+1	−5	−5	−1	+7	+1	+5	+6
10	−2	−1	+2	0	−2	−2	+1	−1	+2	−2	0	+5
11	+1	+2	+2	−2	+1	−1	+5	+3	+1	+6	+1	−5
12	−6	0	+1	−5	0	0	−1	+2	−1	+1	−1	+2
13	+1	0	0	+1	−3	+2	+2	−3	+2	−4	+4	0
14	−7	+4	0	−4	−2	−1	−1	−1	−1	+2	+1	−2
15	0	−4	−1	−5	−1	0	−2	+4	−3	+2	0	+3
16	−2	−6	+5	−2	+3	0	0	−2	+7	+4	+6	+9
17	0	0	+2	−2	−4	+2	−3	+4	+2	+4	−2	0
18	−1	+1	−2	−1	−2	−1	−2	−2	+2	−3	−2	−1
19	0	+1	+1	−1	−1	+1	+3	0	+2	−4	−2	−4
20	−3	−4	+1	−2	0	−1	+1	+1	−2	−1	+2	−3
21	0	+4	+4	0	+1	+3	0	+4	+3	−4	−2	+3
22	−2	+1	0	−2	+1	+3	−1	−1	0	+1	+2	−1
23	+3	−3	−2	−3	+2	−2	+2	−2	+2	+2	+3	+3
24	0	0	−2	+1	0	+3	−3	+4	+1	+2	0	0
25	0	−1	+1	0	−3	+1	+2	+1	0	−1	−3	−1
26	+1	0	0	+2	+1	+2	+1	−1	0	−1	+2	0
27	0	0	0	0	−1	−2	−1	−5	+3	+1	−2	+1
28	+7	−2	+3	+3	+3	−3	+3	+1	0	−3	−2	+2
29	−2	−2	−2	+1	+5	−2	0	+2	+1	+4	+3	0
30	−1	+3	−4	+1	+2	−1	+1	−1	−4	+3	−6	0
31	0	+1	0	−2	+2	−1	+3	−3	+1	−1	−1	0
32	+2	+1	0	+2	0	−1	0	−2	−1	+1	+1	+2
33	+1	0	0	+1	−2	0	0	+2	+1	+1	+1	0

Period.	55°8.	55°8.	69°10.	69°10.	28°4.	28°4.	49°7.	49°7.	64°9.	64°9.	sin 4D.	cos 4D
86	+2	−2	+2	−1	+3	0	+2	−1	+2	−1	0	−2
87	+1	−2	+4	+1	−2	−4	−4	−3	0	−4	+2	−3
88	+1	−1	−2	+1	+3	+3	+4	+2	+5	+1	−1	+4
89	+4	+3	−2	−5	+6	−4	+4	−6	+2	−3	+1	+2
90	+2	+2	+2	−1	+1	−3	−1	−3	0	+1	0	−1
91	−1	+2	−1	0	−1	−3	−2	−2	+1	−1	−1	−1
92	0	+3	−3	0	0	−1	−1	−1	0	−2	−2	−2
93	−2	0	+3	0	−1	−3	−2	−2	0	−3	+2	−3
94	−3	−3	−2	+3	−1	+2	0	+2	0	+1	−1	0
95	−2	−1	−2	−1	+2	+1	+2	0	−1	+1	+1	−1
96	+2	−1	+1	0	+1	0	+1	0	−3	+1	+3	0
97	−2	−1	+2	−1	+2	0	+2	−1	+3	+3	+3	−4
98	−4	+1	+5	+1	+1	+2	+2	+2	−1	+4	0	+3
99	−1	+1	+2	+1	+2	0	+2	−1	+2	−1	+1	−1
100	−2	0	−2	0	+3	−1	+2	−3	−1	−1	+1	−1
101	+4	0	+2	+3	−1	−1	−1	−1	−1	+1	0	−1
102	+2	−1	−1	+3	0	−2	−1	−1	0	+2	+1	−2
103	0	−3	−1	+2	0	+2	+1	+2	+1	0	0	−1
104	+2	−1	0	−2	+1	−1	+1	−1	−4	−1	−4	+1
105	+2	−2	0	−1	+3	+1	+3	−1	0	−3	+2	−3
106	−2	+4	−4	−2	0	+1	0	+1	+1	−2	+1	+2
107	+3	−4	−1	+4	+1	−1	+1	−1	0	0	+1	0
108	+2	+5	+2	−5	−4	−4	−5	−2	+1	0	−1	−1
109	−2	−6	−4	−2	+2	+3	+3	+2	−3	−2	−3	+2
110	+4	+1	+3	+2	−5	−1	−5	+2	−3	0	−2	+1
111	+3	−2	+2	+1	+2	+1	+2	0	−5	+3	−5	+1
112	+2	−1	−2	+1	−4	+1	−3	+4	+9	+4	−6	+8
113	+1	−3	−3	0	−3	+2	−1	+3	−2	−3	+3	+2
114	+2	−2	+1	−5	+4	−1	+3	−2	−6	0	+4	+4
115	+2	−1	+1	+1	−5	0	−5	+3	−2	−4	−4	C
116	+7	+10	−11	−1	−2	+7	+1	+7	−3	+1	−3	+1
117	+1	0	0	−1	−1	0	−1	+1	−4	0	−3	−1
118	−2	+2	0	+4	+1	−2	0	−2	+2	+2	−2	C
119	−2	−4	−4	−3	0	−2	0	−2	+2	+1	−2	−1
120	−1	+3	−2	0	+3	+4	+4	+2	+2	−2	−3	−1
121	+1	+1	0	0	0	−2	0	−2	+1	−2	−2	−3
122	+3	0	−2	−1	+2	−2	+1	−3	−1	−2	−1	−3
123	−6	0	−2	+5	−5	+2	−4	+3	−1	−4	+2	−3
124	−1	0	−2	0	0	−3	−1	−3	0	−2	+2	C
125	0	−5	+5	0	−1	0	−1	+1	+1	+1	−1	−2
126	−3	−1	+3	+1	−1	−1	−1	0	−1	−2	−1	+2
127	−3	+1	+1	+2	−1	+2	0	+2	+2	−1	0	−2
128	0	−3	−2	−4	−1	−2	−2	−1	−2	−1	−2	−
129	−1	0	0	+1	0	+2	+1	+1	+2	−4	+3	
130	−4	+2	−2	−2	−2	−3	−3	−2	0	−1	+1	+
131	+2	0	−2	0	−1	−2	−2	−1	0	−1	+1	+
132	+1	−1	0	0	0	+1	0	+1	+1	0	0	−
133	0	−1	−2	−1	+1	0	+1	−1	0	+1	0	+

eriod.	50°7.	50°7.	29°4.	29°4.	37°5.	37°5.	68°8.	68°8.	77°9.	77°9.	69°8.	69°8.
86	+2	+2	−3	0	+3	−5	+7	−4	+6	−2	+4	+6
87	−2	+4	+5	0	−1	−5	0	+2	−4	−1	+4	+3
88	+2	−3	−1	−5	+4	+3	+1	−2	−1	+1	−2	0
89	0	−1	−1	−6	−1	+1	+2	+2	+3	+1	−2	+1
90	−2	0	−1	+2	−4	+2	+1	+3	−4	+2	+6	+2
91	−2	0	−1	0	0	0	0	−2	−1	+3	0	+1
92	−3	−1	+1	+2	−1	+3	0	+3	+1	+3	−1	−2
93	+1	−3	+1	+2	0	+1	0	−3	+2	+1	+3	0
94	−1	−1	−1	0	+1	0	0	+5	+2	−4	+1	+2
95	+2	−1	−1	+2	+2	+6	−1	−1	−3	−3	0	+5
96	0	+3	−1	−2	+1	−1	−1	−1	0	−1	−2	+1
97	+2	+4	+2	0	+1	−3	−4	+2	−1	−2	0	−4
98	−3	0	−1	−1	−2	−1	0	−1	0	+1	+1	−2
99	0	+2	−5	−4	−3	0	+2	0	+5	0	+5	0
00	−1	0	−1	0	+3	0	−1	+3	−2	−3	+2	+1
01	0	+1	0	0	−1	+2	−1	−1	+1	+1	+2	−2
02	−2	0	+1	−1	−3	−1	−1	0	−1	+1	0	−1
03	−1	0	−1	+3	+1	0	+2	0	0	+4	+3	−2
04	−2	+3	+1	+3	+1	−2	−2	0	+1	0	+2	+1
05	+2	−4	−1	+3	+2	0	−3	+2	+4	+5	−5	+4
06	−1	+3	+1	−4	+1	+1	+3	−1	+2	0	−2	−1
07	+1	+1	+2	+1	−1	−2	+2	+3	−3	−3	−1	0
08	+1	−1	+1	+4	+1	0	−2	−1	−3	+1	+2	0
09	−3	−4	0	+1	−3	−2	+5	−1	+4	−1	−2	−4
10	0	−1	+5	0	+3	+6	+2	−4	+1	+3	0	+1
11	+3	−3	+3	−1	+3	−1	−4	−2	0	+5	+2	−3
12	+6	−6	+1	0	+3	−2	+4	0	+8	+1	+9	−2
13	−2	−3	0	+1	+3	0	−4	+1	+3	−2	−2	+3
14	0	−5	−4	+5	0	+8	+6	0	−4	−5	−3	+3
15	0	+3	0	−2	−3	−4	−1	0	−2	0	−1	−2
16	0	+2	−1	0	0	+3	+2	0	+1	+1	−1	−1
17	−3	0	−3	0	+3	+1	+1	−2	0	+2	+1	0
18	−3	−1	+4	+1	+1	+2	+1	−4	−3	−2	+1	−2
19	−1	−1	+1	0	−3	+2	0	−4	+2	−4	−4	+4
20	−1	−2	−2	+2	0	+1	+2	+1	−6	0	−6	+1
21	+2	−3	−1	0	−1	0	−1	−3	−5	+1	+2	−3
22	+4	0	0	−1	0	−2	−1	+2	−2	+4	+4	0
23	+1	+2	+1	+1	−2	+4	0	0	0	+2	−3	0
24	−2	−1	+5	−2	−1	−1	+1	+2	+2	−1	−1	0
25	−1	+3	−5	+3	+5	0	0	+1	−1	+1	0	+1
26	+2	0	+2	−1	−1	+1	−2	−3	+3	−1	+1	+2
27	−2	+1	+4	0	−1	0	+1	+2	−1	−2	+2	+2
28	−1	+1	0	−1	+1	+3	−4	−5	−6	−3	−3	−2
29	+1	−2	+1	−3	+1	+1	+2	+1	−2	+4	−2	−4
30	+2	0	+2	−1	+1	−1	−1	+1	+2	0	0	−1
31	0	+2	−1	+2	−1	−1	+3	−2	+1	−3	+2	+1
32	+1	0	+4	−1	+1	−2	+1	+2	−1	+2	+1	0
33	0	0	0	−1	+1	+2	0	0	−1	0	−1	+1

Period.	53°6.	53°6.	27°3.	27°3.	45°5.	45°5.	64°7.	64°7.	55°6.	55°6.	37°4.	37°4.
86	+5	0	−1	+4	−1	+4	−2	−5	−2	+4	−1	0
87	0	−1	0	+2	0	+2	−4	+1	0	+4	0	+1
88	−2	−1	0	−2	0	−2	−6	−3	−6	+1	+3	0
89	−2	+1	+4	−2	+4	−2	−4	+5	−4	+4	+2	+4
90	+2	−4	−3	0	−3	0	0	−4	+2	−3	+2	+1
91	+4	−4	0	+2	0	+2	+3	+3	−2	+3	+2	0
92	−2	+4	0	+2	0	+2	+3	+2	−3	+1	−1	−2
93	−3	+1	+3	−1	+3	−1	+1	0	−1	+2	−3	−1
94	−2	−6	+2	−2	+2	−2	+2	0	−1	−1	−1	0
95	+2	−4	+3	−2	+3	−2	−2	−1	−1	+1	0	−1
96	+3	−3	−4	−2	−4	−2	+3	−1	+1	−2	0	−1
97	0	+4	−5	+2	−5	+2	0	0	0	−1	−2	−2
98	0	+3	−5	+2	−5	+2	0	+1	−1	−1	+4	+1
99	−3	+2	−3	−1	−3	−1	+1	+1	−1	+1	−1	+3
100	−1	+2	−5	0	−5	0	0	−4	+4	0	+1	+3
101	−1	−1	−1	+2	−1	+2	+1	+2	−4	−2	+2	+2
102	+4	−4	+3	0	+3	0	+1	0	−1	0	−2	−1
103	+2	0	+3	−4	+4	−4	+2	−1	−2	0	0	−2
104	−1	+2	+2	−3	+2	−3	+2	−1	−1	−2	−3	0
105	−5	+10	−2	0	−2	0	+3	+1	+1	−2	−2	0
106	−4	+3	−1	0	−1	0	+2	−3	+1	−3	−4	+1
107	−2	−2	−2	−1	−2	−1	+2	−2	+3	−1	+2	−4
108	+2	+3	0	−3	0	−3	−5	−4	+1	−6	+4	−1
109	+2	+2	+3	+4	+3	+4	−1	+2	−2	−1	+2	+5
110	−3	+1	+3	+6	+3	+6	+2	−3	+1	+5	−1	+9
111	−2	+2	0	+1	0	+1	0	+6	+3	−8	0	+6
112	−5	+2	+1	−7	+1	−7	+3	−2	−4	0	+1	+4
113	+1	+1	−2	+1	−2	+1	−1	+2	+3	0	+1	+3
114	0	−1	+1	−8	+1	−8	0	−7	−6	−4	−2	+6
115	−3	−2	−1	−6	−1	−6	−2	+1	−2	+1	−1	−2
116	−4	−4	−13	+2	−13	+2	−2	−9	+1	−8	+4	+3
117	−4	+1	+2	+1	+2	+1	−2	−1	0	−2	−2	C
118	−3	−1	+3	+6	+3	+6	+5	−2	0	+4	−1	C
119	+3	−4	+1	+2	+1	+2	0	−1	0	+2	+2	−3
120	+3	−3	+1	0	+1	0	0	+1	0	−1	0	−3
121	+1	+3	+4	0	+4	0	−2	−1	0	+2	−1	+1
122	−7	+1	+2	−2	+2	−2	−2	+3	+2	+2	0	+3
123	−3	−5	+1	−2	+1	−2	+2	−3	−2	−4	−2	−5
124	+5	+3	+2	0	+2	0	+2	+1	+2	+2	+2	−4
125	+3	+1	0	−1	0	−1	−1	+2	−2	+1	0	C
126	−3	+4	−3	+1	−3	+1	−4	+2	−3	−3	0	C
127	−2	+4	+2	−1	+2	−1	0	−3	+3	+1	−1	−1
128	−4	−1	+3	+1	+3	+1	0	−2	0	+2	0	+1
129	+2	−3	+4	−1	+4	0	−1	+2	+1	−1	+1	−2
130	+4	+1	+7	−2	+7	−2	+5	−4	−5	−1	−2	+3
131	+1	+4	−1	−5	−1	−5	−1	+1	+1	+1	+1	−1
132	−1	+2	−1	−4	−1	−4	+1	−1	0	−1	0	+2
133	0	+1	−4	+1	−4	+1	−1	+1	−1	+2	+1	+3

Period.	28^s3.	28^s3.	sin 3D.	cos 3D.	77^c8.	77^c8.	29^s3.	29^c3.	68^c7.	68^c7.	39^s4.	39^c4.
86	−5	+1	−2	+5	−1	+3	−1	−1	−1	0	−1	+2
87	0	+1	+3	+1	0	0	−1	−1	+1	+3	+1	+3
88	+2	+1	+6	+1	+1	+6	+5	−1	+3	−3	+3	+2
89	−3	+1	−1	−3	+1	+5	+3	+2	+2	+2	−3	0
90	0	−1	0	−1	+3	−1	+5	+1	+1	+5	−2	−5
91	+1	+2	+1	−1	−2	+2	−2	−1	+3	0	−2	−2
92	+2	+4	+5	0	0	+1	0	0	−1	−1	−1	+1
93	0	+3	−1	0	+1	−3	−2	+3	+2	+3	+4	+1
94	+2	0	−3	+2	−2	+1	+1	+1	+1	+1	+1	+1
95	0	+1	−1	+1	+2	−1	+2	−1	+1	+1	0	+1
96	0	−2	0	−1	+3	+1	+1	+3	−3	−3	+4	0
97	+2	−3	−4	+2	+1	+4	−5	−1	+4	+5	−4	−5
98	+1	−4	−3	−3	−1	−3	+1	+1	0	−1	−1	+1
99	−3	+2	+2	0	+2	−2	−3	−1	−3	−2	−3	+2
100	0	−2	+1	+2	0	+2	+1	+1	0	0	−1	0
101	+1	+1	−1	+3	+2	−3	+1	−3	0	+3	−2	+2
102	+1	−2	0	+2	0	+3	−3	+1	+1	0	0	0
103	−3	0	+2	+2	+1	−2	+1	+2	+2	−1	−1	+2
104	−1	−2	+2	+2	+1	0	−1	0	−2	0	+2	+1
105	+3	−2	−3	+4	+7	0	+1	−6	+4	+2	+3	0
106	−1	+5	+2	0	+3	−1	+2	−2	−2	+2	0	+2
107	+4	0	−2	0	+3	+1	+2	+2	0	−3	0	−2
108	−3	+4	+1	−4	0	+2	−2	−1	−2	0	−1	−2
109	0	+3	0	−6	−3	0	+1	+1	−2	+2	−1	−4
110	−5	−4	−2	−1	−4	−1	+3	+4	−4	+3	+2	−4
111	0	0	0	+4	−1	0	+1	+2	−2	−2	+1	+3
112	+1	+3	0	−5	+8	0	+8	0	−2	−7	−7	0
113	−1	−2	+1	−2	−3	+1	−3	−2	−4	+1	−3	+2
114	+2	+4	−1	0	0	+2	−3	−1	−5	+4	−1	−6
115	+4	0	+4	+4	+1	−6	−1	+5	−3	−3	+3	−2
116	−2	−3	+2	+2	+5	+5	+2	−7	0	+7	−5	−3
117	+1	0	−3	0	+2	+1	+1	0	−1	−1	0	0
118	−4	+1	+5	−2	0	+1	−1	0	−2	0	0	+1
119	−3	−3	−1	−1	0	0	+1	0	+1	0	0	−1
120	+2	0	−1	0	−1	−1	+1	+1	0	0	0	0
121	−1	0	+2	+5	+3	−4	−5	+1	+4	0	+2	+3
122	−1	+5	+3	+1	−1	+2	+1	+1	0	0	0	0
123	+5	−3	+1	+3	−2	+4	0	+6	+5	+5	−7	−2
124	−5	−3	0	−2	−1	0	−1	−1	+1	−1	−2	0
125	+2	−1	+4	+1	+2	−4	+3	+3	-3	+1	0	+2
126	+3	−1	+1	−3	+1	+1	−1	0	+2	0	+1	−1
127	0	+2	+4	−1	0	−6	−6	+2	+3	+5	+1	+5
128	−2	+1	+2	−2	−3	0	−3	+3	−2	+4	−4	−1
129	−3	0	−1	+1	+1	−1	+1	0	0	0	0	−1
130	−2	+2	+4	−3	+1	−1	0	0	0	−1	−1	+1
131	0	+3	+1	−1	0	0	−1	0	+1	0	−1	−1
132	+1	0	0	0	+1	+1	0	−1	−1	0	−1	0
133	−2	+1	0	+1	0	0	0	0	+1	0	+1	0

Period.	49°5.	49°5.	69°7.	69°7.	50°5.	50°5.	49°4.	49°4.	37°3.	37°3.	50°4.	50°4.
86	+1	+5	−7	+1	+6	+2	+3	− 5	−1	−6	+3	−2
87	−1	+5	−2	−5	−2	−6	−1	− 2	−1	−3	+1	+2
88	−4	0	−1	+3	−5	+4	−2	−10	+9	+3	+1	−3
89	+2	+2	+3	+3	−3	+1	+2	+ 4	−1	−3	+1	0
90	−5	+2	−4	−3	−2	0	0	− 2	−3	+2	+5	+2
91	−2	−1	+2	0	0	0	−1	0	−1	+1	−3	+3
92	−2	−1	0	+2	−1	+1	−2	− 2	+1	−2	−1	−1
93	−3	+2	−1	+3	+2	−1	+2	− 2	−1	+3	−1	+1
94	+1	−1	+1	0	−2	−1	−1	+ 3	0	−1	+6	−1
95	0	0	+1	−1	−3	−2	+2	+ 1	+2	0	+2	0
96	+4	0	−2	−2	−1	+1	+5	− 5	+5	−4	+3	+1
97	+3	−5	−4	−3	−1	+2	−1	− 1	+2	−1	−5	+1
98	0	−1	+1	0	+1	−4	−1	− 2	−1	+2	−3	−4
99	+4	0	0	+4	+2	+4	+1	0	−1	−1	+4	+1
100	−1	0	+2	−1	−2	−1	−1	− 3	−3	−1	−3	0
101	−2	0	−1	+1	−1	−4	+2	− 1	+1	+2	−1	−3
102	0	+1	+1	+1	0	0	−1	0	+2	0	+2	−2
103	0	−2	+2	0	+1	−1	−1	− 2	0	+3	−3	−1
104	0	−2	−1	+1	−2	−1	−1	− 1	−3	+1	0	+4
105	+1	−4	−4	−3	0	0	−3	− 2	−3	−4	0	−3
106	−1	+1	−2	0	+1	0	0	+ 1	−2	+1	+4	0
107	+1	0	0	−1	−3	+1	+1	+ 1	−1	+1	−2	+4
108	−1	+1	+2	−1	−1	+2	−1	− 3	−2	−1	−1	0
109	−6	+2	+4	+6	+2	0	−1	− 2	−3	0	−2	+4
110	−2	−4	−6	+1	+2	−4	+4	− 2	+4	0	+3	+2
111	−1	+1	0	0	+4	0	−2	+ 1	−1	−1	+1	+3
112	+4	−3	−3	+3	+2	+2	+2	0	−3	−2	+1	−4
113	+3	0	−2	−1	−1	−1	0	− 4	−2	−1	+2	−1
114	−8	+1	−5	+7	+4	+4	+2	− 5	+3	−3	+2	−1
115	+3	−1	+1	+2	0	−4	−1	− 2	+2	0	−1	−2
116	−1	−6	+5	+1	−1	−4	+5	+ 6	−6	−4	−6	+1
117	0	−1	−1	+1	0	−2	+3	− 3	−4	+1	−2	−4
118	+1	−1	0	0	−1	+3	−1	+ 1	−1	+1	−2	0
119	−2	0	−2	+1	+4	−1	−4	− 3	0	−4	+3	0
120	+1	−1	+1	+2	−2	+2	0	+ 4	−4	−2	+1	+5
121	−3	+2	+2	−1	+1	0	−1	− 2	+1	+2	−1	+2
122	+2	+1	+1	−2	−1	+2	−1	+ 1	+1	+2	0	−2
123	+3	+5	+2	+3	−1	−1	+4	+ 3	+4	+3	+4	+1
124	−2	+1	−1	−3	−1	+3	+4	− 1	+1	+4	−2	+1
125	−1	0	+2	+2	−2	+3	−1	− 3	0	+2	−3	+3
126	+2	+1	0	−2	0	0	−1	0	0	0	+3	0
127	−2	−4	−2	−1	−3	−2	+2	− 5	−2	−7	−6	+6
128	−1	+4	−3	0	−1	−1	+1	− 2	+2	−1	+1	−2
129	0	−1	+1	+1	+1	+2	+1	− 2	+1	+3	0	−1
130	−1	+1	+1	0	0	−1	−2	+ 1	+2	0	0	−1
131	+2	0	+2	−1	−2	−2	+1	+ 2	+2	0	0	−2
132	+1	+1	−1	+2	−1	−1	+3	+ 1	+2	+1	+1	+1
133	0	−1	+1	0	−1	0	0	+ 1	−1	+1	−2	−1

Period.	64^s5.	64^c5.	77^s6.	77^c6.	39^s3.	39^c3.	53^s4.	53^c4.	27^s2.	27^c2.	68^s5.	68^c5.
86	+4	+5	+6	+1	+4	+1	−3	0	+1	−4	− 6	+ 2
87	−1	0	−1	0	0	+1	−1	−1	−1	+1	− 2	+ 2
88	+3	0	+2	+1	−2	−4	−5	+2	+2	−2	+ 5	0
89	+3	−3	+4	+1	−2	+2	−1	0	+1	−3	+ 2	+ 1
90	+3	0	+1	+2	−1	−6	−4	−4	−1	+2	0	− 3
91	+4	+2	−2	+3	0	+4	−5	−5	+3	+2	− 1	− 2
92	−2	−3	+4	+1	−4	−4	−2	−3	−6	−2	− 8	0
93	+1	+1	−1	−1	+1	0	−2	−2	0	+3	− 2	+ 3
94	+5	+2	−4	−2	0	−4	+3	−1	−1	−3	+ 3	+ 3
95	+1	+1	0	−2	−1	+2	+3	+3	0	+1	+ 3	0
96	−1	0	+1	0	+1	−3	+1	+5	0	−2	− 1	− 3
97	+3	−6	−4	−4	−2	+3	−9	−1	−8	+4	− 7	+ 2
98	+1	0	+1	0	+3	+3	−2	−2	+4	−3	+ 3	+ 4
99	+1	−2	−1	−1	+1	−6	+6	−4	−1	−5	+ 4	+ 5
100	+1	−2	+1	−2	−4	+1	0	+2	0	+4	+ 5	+ 1
101	0	+1	−1	0	+1	−2	+2	+2	+3	+2	+ 4	0
102	0	0	0	−1	−1	+2	+1	+1	−3	−2	− 2	− 5
103	−1	+1	−1	−2	−2	−2	+2	0	+3	+3	− 5	− 1
104	0	−1	+1	−1	+1	+4	0	+5	+3	−1	− 3	+ 1
105	+4	−5	−2	+5	+4	−1	0	+4	−6	0	− 1	+ 6
106	+3	+1	−2	−2	+1	+1	−1	+5	+2	+2	+ 3	+ 1
107	0	+3	+2	−2	0	0	0	+2	−2	−9	+10	− 4
108	−1	0	0	+1	−1	+3	−1	+1	−7	+5	+ 6	− 8
109	−1	−3	−2	0	+1	+1	+1	+6	+6	−2	− 3	− 4
110	+1	−2	−2	−1	+2	+6	−6	−5	−3	−8	− 7	− 4
111	+1	−3	0	+2	−2	+2	−5	+2	−1	+4	− 4	+ 2
112	+2	+1	+3	0	−8	0	−1	+8	+2	−9	+ 7	+ 9
113	−1	+3	−2	+2	0	+1	−2	+4	−2	+5	+ 4	− 4
114	0	0	0	−1	+2	−3	−3	−3	+8	−5	− 5	−11
115	−5	+1	−2	−6	−8	+3	+4	−2	−8	−1	−10	− 1
116	+5	−1	−2	+4	−4	−1	−6	−3	+9	+6	− 3	+12
117	0	−2	+2	+1	−3	+1	+2	−3	−2	−7	+ 8	+ 5
118	−3	0	+4	−1	+5	−6	+7	+2	−1	+2	+ 5	− 3
119	−2	0	+2	0	−1	+4	+3	−1	+4	−3	− 3	− 7
120	−3	+2	+4	+1	+1	−7	+1	0	−9	+2	−10	− 2
121	−2	+1	+2	+2	−1	+5	0	+3	+4	0	− 1	+ 6
122	+1	+3	+4	+1	−3	−4	−1	+1	−6	−5	+ 6	+ 5
123	−3	+1	−2	+3	−2	+8	−6	+3	0	+2	+ 4	+ 2
124	+2	−1	+1	−1	0	−3	−4	+3	0	−2	0	− 4
125	+2	−3	+1	−2	+2	+4	−7	+1	0	0	− 1	− 3
126	+4	+5	−2	+6	−4	+3	+3	0	+3	+2	− 2	+ 1
127	+3	+1	0	+3	−2	−4	+2	0	+2	+2	− 1	− 4
128	0	+2	−2	0	−1	+2	−3	+3	−5	−5	− 7	+ 3
129	+2	−2	0	+4	−2	−3	−2	+2	+1	+3	+ 1	+ 3
130	0	−3	−3	+4	+1	+2	−3	0	+1	−5	+ 7	+ 1
131	0	0	0	−1	+4	−3	−6	−4	−1	+6	− 2	− 8
132	+1	0	0	0	−1	0	+2	−1	−2	−3	− 3	+ 3
133	+1	0	+1	−1	0	−1	−1	−2	−3	0	− 2	+ 3

Period.	41^s3.	41^c3.	55^s4.	55^c4.	69^s5.	69^c5.	28^s2.	28^c2.	sin 2D.	cos 2D.	29^s2.	29^c2.
86	− 4	− 6	+ 2	− 5	+ 1	− 4	+ 3	− 1	− 5	+ 3	+ 1	− 4
87	0	− 3	+ 3	+ 3	0	+ 4	+ 1	+ 4	− 5	+ 3	− 5	+ 3
88	− 6	− 1	+ 5	+ 3	− 3	+ 4	+ 3	− 5	− 5	− 6	0	+ 3
89	− 1	− 3	− 1	+ 1	− 1	− 1	+ 4	+ 2	+ 3	+ 7	+ 4	+ 6
90	− 3	− 1	− 2	+ 2	0	− 2	− 5	− 1	+ 3	+ 10	− 4	− 5
91	− 1	0	− 2	0	+ 2	− 1	0	+ 2	− 1	+ 4	+ 5	+ 2
92	− 7	− 4	+ 2	− 8	− 7	+ 3	− 3	+ 5	+ 2	+ 2	− 1	+ 2
93	− 4	− 1	+ 3	− 3	− 4	− 3	+ 3	− 4	+ 3	+ 3	+ 1	− 2
94	− 4	0	+ 3	+ 2	+ 2	0	+ 3	+ 1	+ 5	− 1	+ 2	+ 1
95	− 4	− 1	− 3	+ 5	− 2	+ 5	− 5	− 1	+ 3	0	0	0
96	− 3	+ 2	− 3	+ 3	− 4	+ 1	+ 3	+ 3	+ 3	+ 2	+ 1	+ 2
97	+ 1	+ 6	− 1	+ 5	− 4	+ 2	− 1	+ 2	− 3	+ 2	+ 1	− 5
98	+ 4	+ 2	− 2	+ 2	− 2	− 1	0	− 1	0	0	+ 1	+ 2
99	+ 1	+ 8	− 3	− 8	+ 7	+ 4	+ 5	− 3	− 1	− 1	− 3	+ 2
100	+ 2	+ 6	+ 3	− 5	− 3	+ 5	− 4	− 2	− 2	− 2	− 1	− 2
101	− 2	+ 3	+ 2	+ 3	0	− 3	− 2	+ 2	+ 1	0	0	0
102	+ 2	+ 5	+ 2	+ 5	+ 4	− 4	+ 2	− 3	+ 1	− 4	− 5	− 1
103	+ 3	+ 4	− 2	+ 4	+ 3	+ 3	− 1	− 1	0	− 2	0	+ 1
104	0	+ 3	− 2	− 1	− 2	0	− 1	− 2	+ 2	− 3	− 1	0
105	+ 2	+ 5	− 1	− 4	− 1	− 4	+ 1	0	+ 3	0	+ 1	+ 5
106	+ 2	+ 2	0	− 3	+ 2	− 2	− 2	0	0	+ 1	+ 1	− 1
107	+ 5	+ 10	+ 10	+ 3	− 1	+ 10	− 3	+ 4	+ 2	− 2	+ 1	0
108	+ 2	+ 10	− 2	+ 9	− 7	− 4	+ 4	+ 1	+ 1	+ 3	+ 1	− 1
109	+ 1	+ 5	− 4	− 1	+ 4	0	+ 4	+ 2	+ 4	0	+ 2	+ 2
110	0	+ 8	− 3	− 6	+ 1	+ 6	− 3	− 4	+ 1	− 2	+ 3	+ 5
111	0	+ 3	+ 2	− 3	− 3	0	+ 3	+ 2	+ 4	− 2	0	0
112	+ 9	+ 8	+ 9	− 8	− 7	− 8	− 1	− 5	+ 5	− 4	0	+ 2
113	+ 5	0	+ 5	− 1	+ 3	− 4	− 4	0	− 3	0	− 1	+ 3
114	+ 11	− 5	+ 11	+ 6	+ 11	+ 4	+ 5	+ 7	− 1	− 3	− 1	− 3
115	+ 9	− 6	+ 1	+ 11	− 4	+ 10	+ 3	− 6	+ 1	0	− 1	+ 3
116	+ 5	− 11	− 5	+ 11	− 10	+ 1	− 4	+ 2	− 3	+ 6	− 9	− 2
117	0	− 10	− 9	+ 6	− 4	− 9	+ 7	− 2	+ 2	+ 2	+ 1	0
118	0	− 8	− 7	− 5	+ 9	− 1	− 2	− 6	− 1	+ 2	0	+ 3
119	− 3	− 6	− 3	− 6	+ 4	+ 6	+ 3	+ 5	+ 2	+ 4	+ 1	− 3
120	− 6	− 8	+ 5	− 7	− 7	+ 2	− 1	+ 1	+ 7	+ 5	+ 5	+ 3
121	− 7	− 1	+ 5	− 4	− 6	− 2	− 3	+ 2	− 2	+ 1	− 2	− 1
122	− 8	− 2	+ 5	+ 4	+ 5	− 2	− 3	0	− 3	+ 3	+ 1	0
123	− 4	− 4	− 5	+ 5	− 2	+ 7	+ 5	+ 1	+ 3	+ 2	− 6	+ 3
124	− 5	+ 2	− 3	+ 6	− 3	+ 6	− 5	+ 3	+ 5	+ 1	− 1	− 2
125	− 4	0	− 4	− 3	− 1	− 4	+ 1	+ 2	− 1	+ 4	− 3	− 1
126	− 1	+ 1	− 1	0	− 1	0	− 2	0	+ 1	+ 2	+ 3	0
127	+ 2	− 4	+ 1	+ 6	− 5	− 2	+ 5	− 2	0	+ 2	0	0
128	− 4	− 7	− 2	+ 8	− 2	− 8	− 5	0	+ 1	0	− 3	− 2
129	− 3	− 1	− 1	+ 3	+ 2	− 3	+ 2	0	+ 1	+ 1	+ 2	+ 3
130	− 7	− 3	− 7	− 2	+ 2	+ 6	+ 4	− 2	+ 3	+ 2	− 1	− 1
131	− 6	+ 7	− 9	0	0	+ 8	0	+ 4	− 4	+ 1	− 4	− 1
132	+ 1	+ 5	− 5	− 1	− 4	+ 3	+ 3	− 2	0	0	+ 1	0
133	− 1	+ 4	0	− 4	− 2	− 4	− 5	+ 1	+ 4	− 2	0	− 2

Period.	45°3.	45°3.	77°5.	77°5.	31°2.	31°2.	64°4.	64°4.	49°3.	49°3.	64°3.	64°3.
86	−3	+3	+5	−6	+3	+7	+2	−1	−6	0	−1	−4
87	0	−2	0	+1	-1	−1	+4	+2	+3	+4	−2	−1
88	−3	+2	+1	+5	+4	−2	−3	−1	+6	−1	−2	−7
89	+2	−2	0	+3	+2	+1	+1	+2	0	+1	−1	0
90	−2	+4	0	+2	+2	+1	0	+7	+3	−3	−1	+3
91	−1	−2	0	+2	0	+2	+4	0	0	0	+1	−2
92	0	+6	−3	−2	+2	−1	+3	0	−1	0	+2	−1
93	−7	−3	−4	−3	+3	+2	−1	0	+2	+1	−1	+6
94	−2	−4	−1	+3	+2	−1	0	0	−2	−1	0	+1
95	0	+3	−2	0	0	+2	+1	+1	0	+3	−1	−2
96	−5	−4	−1	0	−1	0	+3	+4	−3	−5	+5	−2
97	0	−1	−2	−2	−1	−3	−4	−4	−2	−2	+1	+1
98	−2	+1	+2	−2	+1	+3	0	−1	−1	−2	+3	+3
99	+3	−3	+3	+1	−3	−1	+1	+2	0	+1	+1	+1
100	0	+2	−2	+2	+2	+1	0	0	−1	−2	+2	+2
101	−1	+2	−1	0	0	+2	+1	+2	−1	+2	+1	−1
102	+1	−1	−1	+1	−1	+1	0	+1	+1	−1	−1	−1
103	+3	−1	+1	−1	+1	0	−3	−1	−2	0	0	+3
104	0	+3	−3	0	+2	−4	−2	−1	−4	+1	0	−1
105	−6	−1	+4	−2	−2	+1	+1	+4	−3	+1	+2	+1
106	+1	−1	+2	−1	−1	−2	+2	0	+2	−2	+2	0
107	+1	+3	+1	+1	+2	0	+2	−2	+3	+3	−3	−1
108	0	0	0	−1	−1	−1	+2	0	−2	−4	−3	0
109	−1	+2	−2	0	−1	-2	0	+1	−3	−2	+1	−1
110	2	−3	−3	+2	−1	−4	+4	−4	−3	5	+3	+3
111	+6	0	+1	−3	−3	+3	+2	-4	+2	+4	−3	−8
112	−1	+7	+4	−1	−3	−5	−1	0	+6	0	−2	−5
113	+1	+1	0	+3	+2	+2	0	+4	−2	−5	−2	−1
114	+2	+3	−3	−4	−3	−5	−3	+6	−2	−1	+1	−3
115	−2	−3	0	−4	+4	−1	−4	+3	−1	−1	−2	+5
116	−6	−4	−7	−1	+4	−5	+4	−2	+1	−3	−1	−8
117	−3	−1	+1	+1	0	0	+5	0	−2	+6	−8	+3
118	−1	+2	+2	+5	+4	−4	−3	+1	−1	+3	+4	3
119	+2	−3	+1	−3	−2	−2	−3	+1	+3	−1	+5	0
120	−4	−2	+1	+3	+1	+2	+6	0	+6	−3	−1	−1
121	+2	+2	−1	+2	−2	0	+4	+2	−5	−1	−1	−3
122	+2	−3	+1	+5	−3	−5	+4	+1	-3	−1	−2	−3
123	+2	+1	+2	−1	−3	+1	−2	+1	-1	0	+5	+2
124	+3	−3	+1	+1	0	−1	−2	−2	−3	−2	+1	−3
125	−1	−1	+1	−1	−1	−1	−1	+1	+5	−2	−1	+2
126	−3	0	+1	0	+2	0	−2	+1	−1	−1	+3	−1
127	0	−1	+3	+1	0	+3	+2	−3	−2	+5	+3	+3
128	+2	−1	0	+3	−2	−2	−1	−2	0	−1	0	−3
129	0	0	+2	0	−2	−1	+1	+2	−1	0	+2	+2
130	−1	−1	−6	0	+1	+5	-1	+1	−1	+1	−3	+2
131	+4	−3	−3	0	−2	+1	0	+2	0	−1	+1	−2
132	+2	+2	+2	+2	+1	+2	−2	0	−2	−2	−1	−2
133	+1	+1	0	0	0	+1	+2	+4	0	−3	+1	−1

Period.	22^s1.	22^c1.	68^s3.	68^c3.	69^s3.	69^c3.	49^s2.	49^c2.	77^s3.	77^c3.	53^s2.	53^c2.
86	+3	−2	− 7	−3	−5	+ 5	+ 2	−4	+ 3	+ 5	− 1	0
87	−1	0	− 4	0	−5	− 2	− 1	+2	− 1	+ 3	− 3	− 2
88	+4	+3	+10	−6	−3	+12	− 2	+1	− 3	+ 6	− 4	+12
89	−3	0	− 2	−2	+2	+ 2	− 4	0	+ 6	+ 4	− 1	+ 5
90	+3	+3	+ 2	−1	0	0	− 4	−3	− 1	− 2	− 2	+ 1
91	−1	+1	0	0	−1	0	− 2	+1	− 4	0	+ 2	− 4
92	−3	0	− 2	+4	−2	− 3	+ 1	+4	+ 5	+ 1	− 1	− 5
93	+1	−2	+ 1	−2	−3	+ 1	0	+1	− 4	− 2	+ 3	− 3
94	0	+1	0	+1	+1	0	− 1	0	− 1	+ 3	+ 3	+ 1
95	−1	−1	0	−1	+1	− 2	0	+2	+ 1	− 3	+ 2	+ 6
96	+4	+2	+ 5	+1	−3	+ 2	+ 1	−1	− 7	− 1	− 2	+ 9
97	+1	−1	0	−1	−1	+ 1	− 7	+1	+ 9	+ 3	− 1	+ 6
98	−5	−4	+ 3	−1	+4	− 1	+ 1	0	− 6	0	− 3	+ 9
99	−1	+2	+ 4	+1	+2	+ 3	− 4	−4	− 4	+ 6	−11	− 4
100	−2	−3	+ 1	−2	0	+ 1	0	−1	+ 6	− 4	− 4	− 8
101	+2	0	0	−1	0	0	+ 1	−2	− 6	− 3	+ 4	− 7
102	−2	0	+ 1	−3	−1	− 3	− 4	+2	− 1	+ 4	+ 6	− 3
103	−2	−2	− 1	−1	+1	0	0	−2	+ 3	− 2	+ 2	+ 1
104	−2	0	+ 1	0	0	− 1	+ 3	−4	− 6	− 8	− 4	+ 3
105	0	0	− 1	−3	−2	+ 2	+ 3	−2	+ 5	+11	− 7	+ 9
106	+3	−3	+ 1	+5	+2	+ 4	+ 1	−6	+ 3	− 7	− 8	− 1
107	+2	−2	− 1	−1	0	0	− 7	−1	− 8	− 2	− 3	− 8
108	−1	−2	− 4	0	+3	0	+ 2	−1	+ 3	+ 6	+ 7	−10
109	+1	−3	− 1	+2	+4	+ 3	− 2	+1	+ 5	−17	+12	− 2
110	−2	+4	− 4	0	−4	+ 2	−10	+1	−10	+ 4	+13	+ 1
111	+2	+1	+ 3	+4	−3	+ 2	− 1	0	+ 8	−10	+10	+ 2
112	+7	−4	− 3	−1	−1	+ 2	+ 4	−1	−10	− 3	− 2	+15
113	−2	0	− 2	−1	−1	+ 3	+ 2	−2	+ 1	+11	− 9	+11
114	−2	+2	0	−3	0	− 2	− 1	+6	+ 5	−17	−24	+ 4
115	−2	+3	0	−1	+2	+ 2	+ 3	+2	−15	+ 6	−16	− 8
116	+1	+7	+ 1	−3	+2	+ 4	0	+5	+13	+10	− 4	−17
117	−7	+1	− 6	+1	+4	+ 6	− 2	−5	+ 6	−13	+ 7	−17
118	−2	−5	− 3	−2	−3	− 2	− 4	−6	− 2	+ 7	+ 8	− 6
119	−2	+4	0	−1	0	0	0	+2	+ 4	− 2	+ 3	0
120	+3	0	+ 4	−1	−2	0	0	+4	− 7	− 4	+ 6	+ 8
121	+4	+1	− 4	+2	+3	+ 4	0	+4	+ 5	+ 7	+ 3	+10
122	−1	0	+ 1	+6	0	+ 5	+ 4	+7	+ 2	− 5	− 6	+ 9
123	−2	+1	+ 3	−2	−1	+ 1	+10	+4	− 4	− 4	−10	− 5
124	−1	−1	− 2	−3	−1	+ 2	+ 2	+1	+ 5	+ 6	− 2	− 5
125	0	+2	+ 2	+2	+3	+ 2	+ 6	−4	− 1	− 6	+ 3	− 6
126	−3	+2	0	−2	+2	0	+ 1	0	− 2	+ 4	+ 2	− 3
127	+4	−1	− 3	−1	+1	− 3	− 1	+2	0	+ 5	+ 1	+ 3
128	−4	+2	+ 3	−5	−5	+ 1	0	−2	+ 4	− 8	− 7	+ 9
129	+2	−1	− 4	0	−1	+ 4	− 2	−1	− 7	+ 4	− 8	+ 2
130	+5	+1	0	+3	−1	+ 1	− 1	−3	+10	− 3	−10	+ 4
131	+1	−2	− 2	+1	−1	− 1	− 1	+4	− 9	− 7	−14	− 3
132	−1	−1	+ 2	0	−1	− 1	− 1	+2	+ 5	+ 6	− 7	− 4
133	+2	0	+ 1	−1	−1	− 1	+ 1	−1	+ 2	− 3	− 2	− 5

Period.	$27^{s}1$.	$27^{c}1$.	sin D.	cos D.	$31^{s}1$.	$31^{c}1$.	$64^{s}2$.	$64^{c}2$.	$68^{s}2$.	$68^{c}2$.	$69^{s}2$.	$69^{c}2$.
86	− 4	+ 4	+8	−12	−8	+2	+5	−4	+3	+3	−3	−6
87	− 2	− 4	+3	− 7	−5	−3	−4	+3	+1	+3	+5	0
88	+ 1	−12	−2	− 7	+2	+1	−3	−2	+4	+2	+2	−2
89	+ 4	− 4	−8	− 6	−4	+4	+4	0	−1	−2	0	−3
90	+ 1	− 1	−5	− 5	−4	+2	−4	+1	+4	+2	−1	+4
91	+ 3	+ 3	−1	− 4	0	−2	+1	+1	−1	+2	0	−3
92	+ 1	+ 6	−4	− 1	+3	−2	+1	−4	+1	+1	+1	−2
93	− 2	− 2	−6	− 3	+2	−1	+1	+4	+2	0	−1	−2
94	+ 1	+ 2	−3	0	−1	−4	−6	+1	0	0	−1	−1
95	− 5	− 6	−4	+ 2	−1	+1	−2	−1	−2	+1	−3	−1
96	+ 9	− 2	−3	+ 1	−4	+1	+1	−1	0	−3	+3	0
97	+ 2	+ 4	0	0	0	+2	−2	+2	+2	+3	−2	−1
98	− 7	− 4	+4	− 2	−1	0	+1	−1	−6	−1	+3	+5
99	+ 9	+ 5	−3	0	−4	−2	+1	+3	+1	+4	+4	−1
100	− 8	0	−1	+ 2	−1	−3	+2	−2	−2	+2	−1	+3
101	+ 6	− 3	0	+ 2	−2	+1	+3	+1	+2	+1	0	+2
102	0	+ 7	− 3	+ 6	+2	−2	0	−2	−1	0	0	0
103	− 1	− 2	−1	− 3	0	0	−1	+2	−2	0	+1	−1
104	− 2	− 1	−2	− 8	+5	−1	0	−1	−2	+5	+5	−2
105	− 5	− 1	−2	−14	−1	−3	0	0	+3	−4	−1	−4
106	+ 5	+ 3	−1	−11	−1	−2	0	+1	+1	+3	+1	+3
107	− 8	+ 3	−2	− 9	+3	0	+1	0	−3	−1	−2	−3
108	+ 8	− 8	−2	− 9	0	+3	+1	0	+2	+3	−2	+1
109	− 3	+10	0	−10	−1	+4	0	+3	+3	+1	−2	0
110	− 7	− 4	+3	− 9	+5	+1	0	+4	+2	+1	0	−2
111	+ 5	0	−2	−11	+6	−6	−7	0	+5	+3	+6	−3
112	−17	+ 3	−2	−11	−2	−1	−3	−1	+1	−2	+2	−1
113	+ 3	−12	−3	− 3	+1	+7	−7	−2	+5	0	+4	+5
114	+ 3	+20	−1	− 4	+7	−2	−2	−5	+4	+4	−5	0
115	−10	− 7	−4	− 6	+1	−2	+1	0	−2	−2	+2	+4
116	+14	+ 1	−2	− 1	+2	−2	−3	+1	0	+3	+3	−2
117	− 6	+14	+3	+ 8	−2	+3	−2	−1	−2	+7	+4	+4
118	− 1	− 8	−3	+ 6	0	+1	+1	0	−4	−3	−4	−4
119	0	+ 1	−1	+ 3	−2	−4	−2	−1	+5	−1	+3	+4
120	− 6	− 3	−5	− 2	+6	−2	+4	+4	−4	−1	+4	−4
121	+ 7	− 6	−3	+ 1	−2	−3	0	+4	−1	+1	+2	−2
122	− 1	+ 9	+1	+ 6	−4	−7	0	−5	−4	−1	−1	+4
123	+ 1	−11	−7	0	−5	+5	+1	−4	−1	+2	0	+3
124	− 1	− 1	−8	+ 6	−4	+8	+4	+6	0	+4	−2	+3
125	0	− 3	−5	− 4	−2	−4	+4	0	0	−3	+2	0
126	+ 2	− 2	+1	+ 1	−1	+2	+2	−1	−1	0	+2	0
127	− 1	+ 3	−6	+ 6	−1	−1	+1	0	+6	+2	−6	−4
128	+ 8	+ 2	−3	+ 5	+1	−3	+1	+3	−3	−4	−4	+4
129	− 2	+ 6	−6	− 3	+2	−1	+1	−1	−1	−4	−3	−2
130	− 4	− 5	−2	− 4	−1	+5	−3	−1	+2	+2	+1	+2
131	+11	0	+6	− 2	−2	−1	+1	+1	−2	−1	+1	−2
132	− 1	+ 5	+1	+ 2	−1	−2	−4	+1	+2	+5	−3	−5
133	− 3	− 5	−5	+ 3	+3	0	−2	0	−1	0	+1	−1

Period.	$37^{c}1$.	$37^{c}1$.	$77^{c}2$.	$77^{c}2$.	sin F.	cos F.	2.2 cos D $+1^{c}1$.	No. of Observations.
86	−7	+ 4	−7	−1	+ 4	+ 6	−11	118
87	−1	− 7	+5	+4	+ 6	+ 7	− 5	134
88	−1	+10	+3	+8	− 4	− 5	+ 3	123
89	+1	+ 3	−2	+2	− 2	+ 5	−10	121
90	+2	0	−1	0	+ 1	+ 4	−14	126
91	−1	− 2	+1	−1	+ 5	+ 2	− 7	130
92	−5	+ 2	+5	−2	− 1	+ 3	0	128
93	−1	− 1	−4	−2	0	+ 3	− 5	109
94	+2	0	−1	0	+ 2	+ 3	+ 1	109
95	+2	− 3	+1	−4	+ 2	+ 1	+ 1	138
96	0	+ 3	−3	0	− 1	− 1	− 4	140
97	+2	− 2	−3	−1	− 3	+ 5	− 2	135
98	+2	− 1	+2	+3	− 6	+11	+ 1	119
99	+1	− 3	+1	+5	− 7	+ 2	+ 1	111
100	+1	0	+1	−2	− 5	+ 3	+ 1	123
101	+1	+ 2	−1	+1	− 3	+ 3	− 3	125
102	−1	− 1	0	+1	− 3	+ 5	+ 6	124
103	+3	− 2	+3	−1	+ 2	+ 8	0	108
104	+1	0	+2	−1	+ 5	+ 6	0	128
105	+7	− 3	+8	+1	− 2	0	− 6	116
106	0	− 1	+2	0	+ 2	+ 3	− 4	121
107	+3	− 1	−2	−3	+ 3	+ 3	+ 1	116
108	−1	+ 3	−1	+1	+ 7	0	− 2	123
109	+2	+ 2	−1	−1	+ 7	0	+ 1	114
110	+2	+ 4	−1	+3	+ 1	0	+ 2	125
111	−2	− 3	−3	+2	− 4	+ 5	− 4	96
112	−1	+ 3	0	0	+ 2	− 1	+ 6	104
113	−2	+ 4	+2	−1	0	0	+ 3	115
114	0	− 3	+2	−3	− 3	− 9	+ 2	93
115	+8	− 1	+3	+4	− 4	− 3	− 5	115
116	−5	+ 3	+3	+1	− 1	− 1	−11	115
117	0	− 6	+2	−7	− 4	− 1	0	113
118	+2	+ 2	−3	−4	− 3	− 3	+ 1	117
119	−3	− 4	−5	−1	+ 1	+ 3	− 3	114
120	−2	+ 2	−1	+1	− 2	− 4	− 7	100
121	+3	+ 7	+4	−3	− 5	− 5	− 6	120
122	+1	− 2	+1	+3	− 5	− 2	− 1	84
123	−2	+ 1	+1	0	+ 1	− 4	− 5	91
124	−4	− 1	−5	−3	+ 2	0	+ 1	114
125	−1	+ 2	+1	−2	+ 2	− 1	− 7	90
126	−1	+ 5	+2	+3	+ 1	+ 2	− 1	134
127	−2	+ 1	−2	−1	+ 5	+ 3	+ 1	120
128	−2	+ 4	0	−1	+12	− 1	+ 3	127
129	+2	− 3	+4	0	+ 9	0	− 5	128
130	+2	+ 4	−1	−3	+ 3	− 6	− 1	112
131	0	0	−1	0	0	− 9	− 1	123
132	+4	− 4	0	+2	− 2	− 1	+ 1	133
133	−3	− 2	−3	+2	0	+ 1	+ 2	124

TABLE VII.

Comparison for Solar Terms in Moon's Latitude of Observed with Theoretical Coefficients.

Ref. No.	Argument.				Coefficient of Hansen's Tables.	Apparent Correction.		Concluded Coefficient.	Brown's Coefficient. *M.N.* lxv. pp. 286–291.	Argument.			
	g.	*g'.*	F.	D.	Sine.	Sine.	Cosine.			*g.*	*g'.*	*ω.*	*ω'.*
8	0	0	1	4	+	1″·19	−0″·11	−0″·03	+ 1″·1	+ 1″·19	5	−4	5	−4
9	0	0	1	2	+	117·24	+0·04	+0·03	+ 117·3	+ 117·26	3	−2	3	−2
0	0	0	1	1	−	5·41	+0·17	+0·40	− 5·4	− 5·36	2	−1	2	−1
0	0	0	1	0	+	18461·65	−0·27	−0·72	+ 18461·3	(+ 18461·48)	1	0	1	0
1	0	0	1	−1	+	4·91	+0·09	+0·33	+ 4·9	+ 4·80	0	1	0	1
9	0	0	−1	2	+	623·71	−0·05	+0·05	+ 623·7	+ 623·66	1	−2	1	−2
8	0	0	−1	3	−	0·35	+0·05	0·00	− 0·3	− 0·35	2	−3	2	−3
7	0	0	−1	4	+	3·68	−0·06	−0·07	+ 3·6	+ 3·68	3	−4	3	−4
2	1	0	1	4	+	0·21	−0·01	+0·04	+ 0·2	+ 0·21	6	−4	5	−4
4	1	0	1	2	+	15·12	+0·01	−0·01	+ 15·1	+ 15·12	4	−2	3	−2
7	1	0	1	1	−	0·67	−0·09	−0·02	− 0·7	− 0·67	3	−1	2	−1
7	1	0	1	0	+	1010·01	+0·17	−0·03	+ 1010·2	+ 1010·18	2	0	1	0
7	1	0	1	−1	+	0·47	−0·16	−0·05	+ 0·4	+ 0·43	1	1	0	1
8	1	0	1	−2	−	166·58	−0·03	+0·04	− 166·6	− 166·58	0	2	−1	2
0	−1	0	−1	3	−	0·30	−0·01	−0·05	− 0·3	− 0·31	1	−3	2	−3
1	−1	0	−1	4	+	6·58	−0·05	+0·07	+ 6·6	+ 6·58	2	−4	3	−4
0	−1	0	−1	6	+	0·09	+0·03	+0·01	+ 0·1	+ 0·10	4	−6	5	−6
7	−1	0	1	4	+	3·00	−0·08	+0·05	+ 2·9	+ 3·00	4	−4	5	−4
1	−1	0	1	3	−	0·21	+0·05	+0·04	− 0·2	− 0·21	3	−3	4	−3
3	−1	0	1	2	+	199·46	+0·04	−0·01	+ 199·5	+ 199·49	2	−2	3	−2
3	−1	0	1	1	+	0·12	−0·04	−0·05	+ 0·1	+ 0·14	1	−1	2	−1
4	−1	0	1	0	−	999·53	−0·19	0·00	− 999·7	− 999·70	0	0	1	0
6	1	0	−1	1	−	0·61	+0·05	−0·04	− 0·6	− 0·59	1	−1	0	−1
5	1	0	−1	2	+	33·37	−0·01	0·00	+ 33·4	+ 33·36	2	−2	1	−2
8	1	0	−1	4	+	0·47	0·00	0·00	+ 0·5	+ 0·48	4	−4	3	−4
6	0	1	1	2	−	1·28	−0·11	+0·07	− 1·4	− 1·27	3	−1	3	−2
6	0	1	1	1	+	0·81	+0·11	+0·01	+ 0·9	+ 0·80	2	0	2	−1
6	0	1	1	0	−	6·48	−0·06	−0·08	− 6·5	− 6·49	1	1	1	0
1	0	−1	−1	2	+	29·73	0·00	+0·02	+ 29·7	+ 29·69	1	−3	1	−2
9	0	−1	−1	4	+	0·41	+0·06	0·00	+ 0·5	+ 0·42	3	−5	3	−4
9	0	−1	1	4	+	0·16	−0·02	0·00	+ 0·1	+ 0·15	5	−5	5	−4
2	0	−1	1	2	+	7·99	+0·05	−0·01	+ 8·0	+ 8·00	3	−3	3	−2
4	0	−1	1	0	+	4·87	+0·01	+0·04	+ 4·8	+ 4·86	1	−1	1	0
6	0	−1	1	−1	−	0·80	−0·15	−0·14	− 0·9	− 0·81	0	0	0	1
5	0	1	−1	2	−	12·14	+0·12	+0·15	− 12·1	− 12·14	1	−1	1	−2
3	0	1	−1	4	−	0·10	−0·09	+0·01	− 0·1	− 0·11	3	−3	3	−4
5	0	0	3	2	−	0·14	+0·01	+0·07	− 0·1	− 0·14	5	−2	5	−2
3	0	0	3	0	−	6·30	+0·06	−0·01	− 6·2	− 6·30	3	0	3	0
4	0	0	3	−2	−	2·15	−0·04	−0·04	− 2·2	− 2·19	1	2	1	2

Ref. No.	Argument.				Coefficient of Hansen's Tables.	Apparent Correction.		Concluded Coefficient.	Brown's Coefficient. *M.N.* lxv. pp. 286–291.	Argument.			
	g.	*g'*.	F.	D.	Sine.	Sine.	Cosine.			*g*.	*g'*.	*ω*.	*ω'*.
					″	″	″	″	″				
6	2	0	1	2	+ 1·52	−0·03	+0·04	+ 1·5	+ 1·52	5	−2	3	−2
25	2	0	1	0	+61·89	−0·05	−0·05	+61·9	+61·91	3	0	1	0
43	2	0	1	−1	+ 0·11	+0·07	+0·01	+ 0·1	+ 0·11	2	1	0	1
65	2	0	1	−2	−15·57	−0·01	−0·09	−15·6	−15·57	1	2	−1	2
82	−2	0	−1	4	+ 0·64	+0·03	−0·01	+ 0·7	+ 0·64	1	−4	3	−4
35	−2	0	1	4	+ 2·42	+0·03	−0·05	+ 2·4	+ 2·41	3	−4	5	−4
78	−2	0	1	2	− 1·62	+0·02	+0·01	− 1·6	− 1·62	1	−2	3	−2
72	2	0	−1	0	+31·76	−0·09	−0·01	+31·7	+31·76	1	0	−1	0
30	2	0	−1	2	+ 2·15	−0·91	+0·02	+ 2·1	+ 2·15	3	−2	1	−2
12	1	1	1	2	− 0·26	+0·05	+0·02	− 0·2	− 0·24	4	−1	3	−2
24	1	1	1	1	+ 0·10	−0·05	−0·02	+ 0·1	+ 0·10	3	0	2	−1
42	1	1	1	0	− 5·33	+0·02	−0·01	− 5·3	− 5·33	2	1	1	0
85	1	1	1	−2	− 7·47	0·00	−0·08	− 7·5	− 7·46	0	3	−1	2
62	−1	−1	−1	4	+ 0·52	+0·10	+0·03	+ 0·6	+ 0·60	2	−5	3	−4
19	−1	−1	1	4	+ 0·35	0·00	−0·02	+ 0·3	+ 0·34	4	−5	5	−4
56	−1	−1	1	2	+ 8·91	−0·06	+0·10	+ 8·9	+ 8·90	2	−3	3	−2
93	1	1	−1	0	− 5·10	−0·02	−0·07	− 5·1	− 5·10	0	1	−1	0
52	1	1	−1	2	− 0·83	−0·05	+0·09	− 0·9	− 0·83	2	−1	1	−2
16	1	−1	1	2	+ 1·16	−0·02	0·00	+ 1·1	+ 1·14	4	−3	3	−2
51	1	−1	1	0	+ 6·76	+0·02	+0·09	+ 6·8	+ 6·76	2	−1	1	0
92	1	−1	1	−2	+ 0·80	+0·01	−0·02	+ 0·8	+ 0·80	0	1	−1	2
57	−1	1	−1	4	− 0·15	+0·01	−0·05	− 0·1	− 0·17	2	−3	3	−4
49	−1	1	1	2	− 1·32	+0·03	+0·07	− 1·3	− 1·32	2	−1	3	−2
90	−1	1	1	0	− 5·66	+0·04	+0·04	− 5·6	− 5·66	0	1	1	0
59	1	−1	−1	2	+ 1·78	+0·05	−0·02	+ 1·8	+ 1·77	2	−3	1	−2
83	0	−2	−1	2	+ 1·10	−0·02	+0·05	+ 1·1	+ 1·10	1	−4	1	−2
36	0	−2	1	2	+ 0·39	−0·04	−0·02	+ 0·4	+ 0·39	3	−4	3	−2
71	0	2	−1	2	− 0·13	−0·08	−0·12	− 0·2	− 0·14	1	0	1	−2
10	1	0	3	0	− 1·02	−0·04	+0·01	− 1·0	− 1·02	4	0	3	0
40	1	0	3	−2	− 0·33	+0·04	−0·03	− 0·3	− 0·33	2	2	1	2
13	−1	0	3	2	− 0·25	+0·01	−0·09	− 0·2	− 0·24	4	−2	5	−2
44	−1	0	3	0	− 2·79	0·00	+0·10	− 2·8	− 2·81	2	0	3	0
86	−1	0	3	−2	+ 0·29	0·00	−0·03	+ 0·3	+ 0·29	0	2	1	2
1	3	0	1	2	+ 0·14	−0·05	+0·04	+ 0·1	+ 0·14	6	−2	3	−2
11	3	0	1	0	+ 3·98	−0·03	−0·05	+ 4·0	+ 3·98	4	0	1	0
41	3	0	1	−2	− 1·52	+0·01	−0·02	− 1·5	− 1·52	2	2	−1	2
89	3	0	−1	−2	− 0·27	−0·04	+0·01	− 0·3	− 0·26	0	2	−3	2
48	3	0	−1	0	+ 1·59	−0·02	−0·06	+ 1·6	+ 1·59	2	0	−1	0
15	3	0	−1	2	+ 0·15	−0·06	−0·04	+ 0·1	+ 0·15	4	−2	1	−2

Ref. No.	Argument.				Coefficient of Hansen's Tables.	Apparent Correction.		Concluded Coefficient.	Brown's Coefficient. *M.N.* lxv. pp. 286–291.	Argument.			
	g.	g'.	F.	D.	Sine.	Sine.	Cosine.			g.	g'.	ω.	ω'.
					"	"	"	"	"				
22	2	1	1	0	−0·64	−0·01	−0·01	−0·6	−0·64	3	1	1	0
63	2	1	1	−2	−0·66	−0·01	+0·02	−0·7	−0·66	1	3	−1	2
38	−2	−1	1	4	+0·22	−0·04	+0·03	+0·2	+0·22	3	−5	5	−4
68	2	1	−1	0	−0·31	−0·06	+0·05	−0·4	−0·31	1	1	−1	0
7	2	−1	1	2	+0·13	0·00	−0·02	+0·1	+0·11	5	−3	3	−2
28	2	−1	1	0	+0·80	−0·09	−0·01	+0·7	+0·81	3	−1	1	0
77	2	−1	−1	0	+0·32	−0·01	+0·02	+0·3	+0·30	1	−1	−1	0
34	2	−1	−1	2	+0·12	+0·09	+0·01	+0·2	+0·13	3	−3	1	−2
84	1	2	1	−2	−0·28	+0·03	+0·02	−0·3	−0·27	0	4	−1	2
60	−1	−2	1	2	+0·32	+0·03	+0·01	+0·3	+0·32	2	−4	3	−2
54	1	−2	1	0	+0·14	−0·13	0·00	0·0	+0·12	2	−2	1	0
95	−1	2	−1	2	−0·10	−0·01	−0·07	−0·1	−0·11	0	0	1	−2
45	−1	2	1	2	−0·12	+0·03	+0·05	−0·1	−0·12	2	0	3	−2
87	−1	2	1	0	−0·11	−0·05	0·00	−0·1	−0·10	0	2	1	0
3	2	0	3	0	−0·12	−0·05	+0·10	−0·2	−0·12	5	0	3	0
69	−2	0	3	0	+0·11	+0·07	−0·01	+0·2	+0·13	1	0	3	0
4	4	0	1	0	+0·26	0·00	−0·03	+0·3	+0 27	5	0	1	0
21	4	0	1	−2	−0·13	0·00	0·00	−0·1	−0·14	3	2	−1	2

TABLE VIII.

Comparison for Figure of Earth Terms in Moon's Latitude of Observed with Theoretical Coefficients.

Ref. No.	Argument.	Coefficient of Hansen's Tables.		Apparent Correction.		Hill's Coefficient.
		Sine.	Cosine.	Sine.	Cosine.	
		"	"	"	"	"
97	g+F−(−☊)	−0·48	...	0·00	−0·08	−0·45
98	F+(−☊)	−0·35	...	+0·05	+0·06	−0·35
99	F−(−☊)	−8·26	+1·66	+0·13	−0·13	−8·73
100	2D−F+(−☊)	...	...	+0·04	−0·03	−0·32
101	−g+F−(−☊)	+0·48	...	+0·03	−0·13	+0·49

fluctuation of the mean error, such as may be introduced by change of refraction, instrumental errors, &c. Multiplication by 2·2 cos D+1·1 instead of by cos D eliminates these fluctuations, and retains any periodicity in the coefficient of cos D such as may arise from a term of the form $g-g'+\omega$, for example. For this reason also it was necessary to investigate the coefficient of sin $(g-g'+\omega)$ with D as the auxiliary angle.

D is, for numerical purposes, taken as ${}_{57}A_2$, or an angle which goes through two revolutions in fifty-seven lunar days. Its epoch is, however, adjusted every period, so as to keep D=0 corresponding to new moon. Every other auxiliary angle has its

mean motion expressed by its notation. Its epoch is such that it passes through the value zero at day 0 or day $0\frac{1}{2}$, according as its first suffix is not, or is, divisible by 4. Day 0 is mean lunar noon (about 9^h G.M.T.) on 1750 September 12.

From columns 53^s2, 53^c2 of Table VI., by a formula suggested by line 70 of Table I., two columns were formed which would approximately represent the mean product of each period of every error by $2 \sin F$, $2 \cos F$ respectively. Before setting these columns down at the end of Table VI., each quantity in column "sin F" was increased by 3 ($0''\cdot30$) in order to bring the mean to approximately zero. Each quantity in column "cos F" (periods 86 to 117) was corrected for Newcomb's corrections multiplied by the ratio of the principal inequality in latitude to the radius. Each quantity was then further increased by $+0''\cdot5$ to make the mean approximately zero, showing that the tabular values of F used during the last fifty years, and corrected before 1883 from Newcomb's corrections, have in the mean been $5''\cdot5$ greater than the observed. This indicates a correction of about $+5''$ to the position of the node, the question of the observed motion of the node being deferred for the present.

These two columns, "sin F" and "cos F," were those used for arguments $F \pm \Omega$ in the figure of Earth terms.

Turning now to Table VIII., it will be seen that Dr. Hill has given too large a coefficient to the principal figure of Earth terms. I am unable also to trace the existence in Hansen's tables of a term with coefficient exceeding $0''\cdot1$ with argument $2D-F-\Omega$. This is very probably an algebraical error of my own ; but it is at any rate quite certain that any coefficient that is employed by Hansen's tables is not in need of serious correction. I have failed, therefore, to verify the term of Dr. Hill's

$$-0''\cdot32 \sin (2D-F-\Omega)$$

I wish to call attention to the term with argument ω (Ref. No. 94, Table VII.). The period of this term is about six years, and the correction to Hansen's tabular coefficient obtained from the observations is confirmed by Professor Brown's theory. I argue from this that the large corrections obtained to the principal figure of Earth terms both in longitude and latitude are real, and are far larger than can fairly be ascribed to change of observers or other observing errors.

Lastly, as to the accuracy of the arithmetic, it will be seen that Table VI. contains two auxiliary angles, ${}_{27}A_3$ and ${}_{45}A_5$, which are identically equal. The process of formation of the columns 27^s3, 27^c3, however, was entirely independent of the formation of columns 45^s5, 45^c5, and it will be seen that the two pairs of columns mutually confirm one another. This agreement, however, was only obtained after the detection of 4 errors in the 192 numbers, consisting of 5, 2, 1, and 1 units respectively. The sum of these errors added without regard to sign and

divided by 192 is 0″·005, a negligible quantity. Although, therefore, the other columns of Table VI. have not been subjected to so searching a test, I believe the results as given in Tables VII. and VIII. may be relied on to within the accidental errors due to the observations.

On the Discordant Values of the Principal Elliptic Coefficient in the Moon's Longitude. By P. H. Cowell.

The values given for the coefficient of the principal elliptic term in the Moon's longitude have varied from 22637″·15 of Hansen's theory to 22641″·6 of Airy's tabular places. In the present note I take some of the values published during the last half-century, and I try as far as possible to trace the discordances to their source. The values in question are—

Ref. No.	Author.	Coefficient.	Material.	Reference.
		″		
1	Airy	22639·06	Greenwich 1750–1851	*Memoirs R.A.S.* xxix. p. 13.
2	Newcomb	·82	„ 1847–1858	Corrections to Hansen's Tables, p. 29.
3	„	·50	Greenwich and Washington 1862–1874	„ „
4	Nevill	·32	Greenwich 1862–1877	*Memoirs R.A.S.* xlviii. p. 417.
5	Cowell	·54	„ 1750–1851	*Monthly Notices*, vol. lxv. p. 147.
6	„	·46	„ 1847–1901	„ „

I shall establish the propriety of the following corrections to the above values :

Ref. No.	Solar Correction.	Planetary Corrections.	Corrected Value.
	″	″	″
1	+0·16	...	22639·22
2	−0·25	+0·06	·63
3	−0·25	+0·10	·35
4	+0·05	+0·06	·43
5	...	...	·54
6	...	...	·46

It will be seen that the accordance of the six values is improved by these corrections. The range is reduced from 0″·76 to 0″·41, or, leaving out Airy's result, from 0″·50 to 0″·28.

One of my values (Ref. No. 5) is based upon the same observed places as Airy's, and the discordance is not therefore due to errors of observation. Either Airy's analysis or mine is wrong.

Airy obtains his result by finding a correction, $-2''\cdot54$, to the value used in his tabular places. As Airy divides his observations into two groups only, each group covering a range of 180° in the mean anomaly, and as the coefficients of some of his terms are over $3''$ in error, an error of 10 per cent. in his correction is intelligible, and in examining the accordance of various results Airy's may therefore be left out of account. Airy, in fact, missed a great opportunity. He had material with which any short-period coefficient could certainly be obtained to within $0''\cdot2$ (see my result, Ref. No. 5),and for 1 per cent. additional to the labour actually expended in the reductions he might have given observed values for all coefficients several years in advance of equally good theoretical ones ; and if he had taken a year, instead of nine years, as his unit of analysis, he could not have failed to anticipate Professor Newcomb's discovery of the *Jupiter* evection term.

I will now explain the foregoing corrections. The solar corrections are intended to reduce the results to what would have been obtained if the coefficients $-8''\cdot44$ and $+18''\cdot55$ had been used for the terms sin $(g+\mathrm{D})$ and sin $(g-\mathrm{D})$. Both Airy and Professor Newcomb have overlooked the necessity of combining a consideration of these two terms with a discussion of the eccentricity of the Moon's orbit. Airy was fortunate in so far that the errors of the coefficients of the terms sin $(g\pm\mathrm{D})$ employed by him are far smaller than those of many of his other coefficients. Hansen had published in the *Darlegung* a large correction to the coefficient of sin $(g-\mathrm{D})$ before Professor Newcomb made his investigations. That the values above quoted for the coefficients in question are final may be inferred from the fact that Hansen's, Delaunay's, and Professor Brown's theories and my own observed values are in close agreement.

In *Memoirs R.A.S.* vol. xlviii. p. 315 Mr. Nevill points out that a term (in the notation of Hansen and Professor Newcomb) $\delta\mathrm{B}$ sin $(g'+\mathrm{D})$ will produce an apparent effect $-\delta\mathrm{B}\times0\cdot70$ sin g'. He arrives at the factor $-0\cdot70$ by supposing that the observations are uniformly distributed from first quarter to last quarter. I employ the factor $-0\cdot48$ obtained as the mean value of cos D. Mr. Nevill nowhere states the point in question in more general terms than in the above special case, but I conclude that he has in fact corrected his result for the error $0''\cdot60$ sin $(g-\mathrm{D})$ in the course of pp. 315–318 of his memoir. The small correction I have therefore applied to his result represents an error $0''\cdot10$ sin $(g+\mathrm{D})$ only.

Coming now to the planetary corrections, results Nos. 1, 5 need no correction, as the observations extend over 100 years. In No. 6 the results extend over fifty-four years, and the planetary terms have already been applied to the individual tabular places. Therefore No. 6 needs no correction either. Also results 5 and 6 cover 150 years between them, and as they are in close agreement, the presumption is that there exists no undiscovered term capable

of affecting the result. Planetary corrections have, therefore, only to be applied to Nos. 2, 3, 4.

(1) $+0''\cdot316 \sin(g+2\varpi-3J+7^\circ)$ calculated by M. Radau, and in *Monthly Notices*, lxv. p. 135, shown to be confirmed by the observations. The term was unknown to Professor Newcomb and to Mr. Nevill.

Ref. No.	Mean Epoch.	Value of $2\varpi-3J+7^\circ$.	θ.	Coefficient multiplied by $\frac{\sin\theta}{\theta}$	Correction.
		°	°	″	″
2	1853·0	60	58	+0·27	−0·13
3	1868·5	270	63	+0·26	0·00
4	1870·0	255	77	+0·22	+0·06

The first line of the above table reads as follows :

Professor Newcomb's investigation (Ref. No. 2) has a mean epoch 1853·0 when the value of $2\varpi-3J+7^\circ$ takes the value 60°. During the period over which the investigation extends, $2\varpi-3J+7^\circ$ varies from $60^\circ+58^\circ$ to $60^\circ-58^\circ$. The correction required is therefore $-0''\cdot316\,\frac{\sin 58^\circ}{58^\circ}\cos 60^\circ = -0''\cdot13$.

(2) $-1''\cdot1 \sin(g+2\varpi-2J)$. This is the *Jupiter* evection term discovered by Professor Newcomb, and attributed to the action of *Jupiter* by Mr. Nevill. Mr. Nevill uses $-1''\cdot4$ as the coefficient, and the correction consequently required by his result is insensible. The coefficient $-1''\cdot1$ is indicated by the observations 1750 to 1901, whereas Dr. G. W. Hill and M. Radau agree in giving $-0''\cdot9$ as the theoretical coefficient.

The term was discovered by Professor Newcomb as a wave of $17\frac{1}{2}$ years from crest to crest in the values of the coefficients of $\sin g$ and $\cos g$, as given by two investigations extending over twelve and thirteen years respectively. Under these circumstances a high degree of accuracy was of course unattainable. Professor Newcomb gives as the empirical term

$$-1''\cdot5 \sin(g+253^\circ\cdot2+21^\circ\cdot6\,(t-1868\cdot5))$$

The means of the values of h, exhibited by Professor Newcomb on p. 28 of his paper, are $+0''\cdot40$ from 1847 to 1858, and $+0''\cdot54$ from 1862 to 1874 ; to these means Professor Newcomb applies corrections $-0''\cdot07$ and $+0''\cdot11$ respectively for his empirical term, thus obtaining $+0''\cdot33$ and $+0''\cdot65$ (p. 29) ; the corrections resulting from the actual term (calculated as in the case of the term $+0''\cdot316 \sin(g+2\varpi-3J+7^\circ)$) are $-0''\cdot22$ and $+0''\cdot03$ respectively. Hence his resulting values of the principal elliptic coefficient require correction on this account,

Ref. No.	Correction.
	″
2	+0·15
3	+0·08

(3) $-0''\cdot68\sin(g+2\varpi+3\mathrm{V}-5\mathrm{E})$. This is a *Venus* term calculated by M. Radau. Its verification from the observations proved very troublesome, as there is a periodic term running through the errors with argument, Moon's longitude, and very probably arising from an erroneous tabular lunar parallax. This term gains one revolution a century upon the *Venus* term, and in *Monthly Notices*, vol. lxv. pp. 136 and 148, I have succeeded in fixing its coefficient at $0''\cdot7$. These two terms were not discovered by Professor Newcomb at the time when he discovered the *Jupiter* evection term, because about the year 1870 the two terms as nearly as possible cancel each other. On account of the two terms I have applied the following corrections :

Ref. No.	Correction for *Venus* Term.	Correction for Empirical Term.	Sum.
	$''$	$''$	$''$
2	$+0\cdot12$	$-0\cdot08$	$+0\cdot04$
3	$-0\cdot11$	$+0\cdot13$	$+0\cdot02$

The corrections are small, partly because the terms nearly cancel and partly because the length of the wave introduced in the values of the coefficients of $\sin g$ is nine years, and is therefore more thoroughly eliminated from a twelve or thirteen year discussion than a seventeen year term is.

It is worth noting that though the two terms cancel in 1870, yet in 1853 their epochs differ by 60°, and the resultant term has a coefficient of $0''\cdot7$, equal to either component term and giving rise to a negative minimum in the values of h at the epoch 1851·6 about. Consequently the values of h, k are not in such good accordance from 1847 to 1858 as they are from 1862 to 1874. This fact Professor Newcomb has evidently observed, but the true explanation, viz. the coexistence of two unsuspected terms of nearly the same period of course could not have been discovered on the evidence. He therefore suspects the earlier observations of being decidedly less accurate, and he writes (p. 29): "I consider that the results of the second series are entitled to three times the weight of those of the first."

Adding together the foregoing three corrections the totals are :

Ref. No.	Correction.
	$''$
2	$+0\cdot06$
3	$+0\cdot10$
4	$+0\cdot06$

These are the quantities applied at the beginning of this paper.

In *Memoirs R.A.S.* vol. xlviii. p. 312, Mr. Nevill writes : "The probable error of a single year's result may be taken as $\pm0''\cdot40$." This would indicate a probable error of about $\pm0''\cdot12$ to the second, third, and fourth results. This is not inconsistent with a discordance of $0''\cdot28$ (decreased from $0''\cdot32$) between

Professor Newcomb's two results. Professor Newcomb's second result and Mr. Nevill's are now in close agreement, as is only natural seeing that Mr. Nevill's sixteen years include the thirteen years of Professor Newcomb's second result. The discordance has been decreased from 0″·18 to 0″·08 by the calculations of this paper.

Note on Diurnal Variations of the Nadir and Level of the Transit Circle at the Royal Observatory, Greenwich.

(*Communicated by the Astronomer Royal.*)

In a former paper communicated to the Society in 1899 March it was shown that when the observations of the level and nadir taken at different times of the same day were compared with each other, those observations made about 6 P.M. showed discordances from those made about midday and midnight. The present paper continues these comparisons of the level and nadir near noon, 6 P.M., and midnight for the years 1897–1904. The results for these times are derived from about 100 days in each year in which one observation at least falls within the limits of each group, viz. 9^h–15^h, 15^h–21^h, and 21^h–23^h civil time. In order to carry the discussion through the whole twenty-four hours, observations made between 3^h–9^h have been compared with those obtained between 15^h–21^h on the same or the previous days. There are approximately thirty days in each year when such observations have been made. The results are given in the following table :

Diurnal Changes of Level and Nadir for the years 1897–1904.

Year.	Level.				Nadir.			
	Noon.	6 p.m.	Midnight.	6 a.m.	Noon.	6 p.m.	Midnight.	6 a.m.
	″	″	″	″	″	″	″	″
1897	+0·20	·00	+ ·13	+ ·30	+ ·11	·00	+ ·17	+ ·33
1898	+0·33	·00	+ ·23	+ ·36	+ ·16	·00	+ ·11	+ ·05
1899	+0·29	·00	+ ·18	+ ·30	+ ·17	·00	+ ·19	+ ·24
1900	+0·29	·00	+ ·23	+ ·46	+ ·14	·00	+ ·05	+ ·02
1901	+0·28	·00	+ ·26	+ ·31	+ ·17	·00	+ ·09	+ ·06
1902	+0·21	·00	+ ·20	+ ·34	+ ·20	·00	+ ·17	+ ·09
1903	+0·14	·00	+ ·22	+ ·32	+ ·04	·00	+ ·09	+ ·06
1904	+0·27	·00	+ ·29	+ ·55	+ ·06	·00	+ ·15	+ ·12
Mean	+0·25	·00	+0·22	+0·37	+0·13	·00	+0·13	+0·12

The variation of the level has a period of 24^h with its maximum about 6 A.M. and minimum about 6 P.M. The variations of the nadir are much smaller, and do not show any conclusive result except the discordance near 6 P.M.

On the Influence of Vapour Pressure on Refraction.

By Dr. L. de Ball.

(*Communicated by the Astronomer Royal.*)

The theory of refraction which M. Radau develops in his excellent memoir, "Essai sur les Réfractions astronomiques,"* concerns also the influence of the pressure of the vapours present in the atmosphere.† Although this memoir appeared quite sixteen years ago, refraction, even to this day, is almost without exception computed without any consideration to the dampness of the air. The consequence of this neglect, however, is that the zenith-distances, thus reduced, contain systematic errors of annual period. Since, so far as I know, attention has not yet been called to this point, a brief dissertation on the real state of the matter may not be quite out of place.

Let ρ be the density of the air at the place of observation, c a constant, and put

$$(1) \qquad \frac{c\rho}{1+2c\rho} = a$$

then the refraction for the apparent zenith-distances $z \leqq 75°$ may be represented by the series

$$(2) \quad \text{Refraction} = \frac{a}{\sin 1''}(a_0 \tan z - a_1 \tan^3 z + a_2 \tan^5 z - a_3 \tan^7 z)$$

The value of the coefficient a varies with the density of the air. If the density of the air have the value ρ_0, and we write

$$(3) \qquad \frac{c\rho_0}{1+2c\rho_0} = a_0$$

then the result is at once

$$(4) \qquad a = \frac{\rho}{\rho_0} \frac{a_0}{1-2a_0\left(1-\frac{\rho}{\rho_0}\right)}$$

If, therefore, the real value for any density of the air be known, the value of a for each density can be computed; and when a is known and the temperature besides, the sum of the above series may also be ascertained. The density is found from the readings of the barometer, of the interior and exterior thermometer, in connexion with the vapour-pressure, with the height above sea-level of the place of observation and its geographical latitude;

* *Annales de l'Observatoire de Paris*, t. xix.

† Older theories, in which the influence of the dampness in the air is at least touched upon, are found mentioned in C. Bruhns' *Die astronomische Strahlenbrechung*. C. Bruhns, however, says in conclusion (p. 173): "Rightly, therefore, have Bessel, . . . paid no further attention to the damp air."

and taking for the unit of density the density of dry air at 0° Celsius, under pressure of one atmosphere (column of mercury 760 mm.), if B is the reading of the barometer (millimetres), τ the reading of the interior thermometer (Celsius), h the height of the point of observation above sea-level, expressed in metres, and ϕ its geographical latitude, then the air-pressure, p, follows from the formula

$$(5)\quad p=(1-0{\cdot}0000001968h-0{\cdot}00265\cos 2\phi)\frac{B}{760}(1-0{\cdot}000162\tau)$$

The air constantly contains a certain amount of vapour. According to the researches of Dalton, the pressure of a mixture of air and vapour, at the temperature t, is equal to the sum of their separate pressures at the same temperature. Therefore, if p be the pressure of the damp air (which can be obtained from the reading of the barometer by the foregoing formula) and if π be the pressure of the vapour contained in the air, then $p-\pi$ is the pressure which the dry air alone exerts. According to the Gay-Lussac–Mariotte law, however, the quotient for dry air

$$\frac{\text{Pressure}}{\text{Density}\times(1+at)}$$

where $a=0{\cdot}003663$ (Regnault) is the expansion coefficient of dry air, is a constant. As for $p=1$ and $t=0°$ C. the density is to be $=1$, so must the above-mentioned constant be equal to 1; hence the equation is for dry air at the temperature t

$$(6)\qquad \text{Density}=\frac{\text{Pressure}}{1+at}$$

If, therefore, we indicate the density of the dry air of the temperature t, and under the pressure $p-\pi$ by ρ_1, we have

$$\rho_1=\frac{p-\pi}{1+at}$$

If the vapour of the temperature t, whose pressure is indicated by π, be replaced by dry air of the temperature t, which exerts the same pressure π, the density of this air, according to equation (6), would be equal $\frac{\pi}{1+at}$. Experiment, however, shows that the weight of a volume of vapour is only 0·622 of the weight of an equal volume of dry air at the same temperature and pressure, and therefore, for like temperature and like pressure, the density of the vapour is also 0·622 of the density of the dry air; and we have for the density ρ_2 of the vapour

$$\rho_2=0{\cdot}622\frac{\pi}{1+at}$$

Now the density of the damp air is equal to the sum $\rho_1+\rho_2$, and writing the previously ascertained values of these we have

$$\rho = \frac{p-0{\cdot}378\pi}{1+at}$$

or if 0·378 be replaced by $\frac{3}{8}$

$$\rho = p\frac{1-\frac{3}{8}\frac{\pi}{p}}{1+at} \tag{7}$$

If the vapour pressure be expressed in millimetres by the height of a mercury column, whose pressure is equal to that of the humidity contained in the air, and if the height of this column of mercury be likewise indicated by π, then it follows from equation (5), and from the corresponding equation for the vapour pressure, that in equation (7) $\frac{\pi}{p}$ may be replaced by $\frac{\pi}{B}$; and from the same equations (5) and (7), with the above assigned value of a, we obtain

$$\rho = \frac{B}{760}\frac{1-0{\cdot}000162\tau}{1+0{\cdot}003663t}(1-0{\cdot}0000001196h-0{\cdot}00265\cos 2\phi)\left(1-\tfrac{3}{8}\frac{\pi}{B}\right)$$

or

$$\rho = \frac{B}{760}\frac{1-0{\cdot}000162t}{1+0{\cdot}003663t}[1-0{\cdot}0000001196h-0{\cdot}00265\cos 2\phi+0{\cdot}000162(t-\tau)]\left(1-\tfrac{3}{8}\frac{\pi}{B}\right) \tag{8}$$

For $B = 760^{mm}$, $\tau = 0°$, $h = 0$, $\phi = 45°$, $\pi = 6^{mm}$, therefore for the density

$$\rho_0 = 1-\tfrac{3}{8}\frac{6}{760} \tag{9}$$

Professor Bauschinger gives for value of the refraction constant $a_0 = 60''{\cdot}15 \sin 1''$. We have

$$\frac{1-\frac{3}{8}\frac{\pi}{B}}{1-\frac{3}{8}\frac{6}{760}} = 1+\frac{3}{8B}\left(6\frac{B}{760}-\pi\right)$$

If we put

$$B[1-0{\cdot}0000001196h-0{\cdot}00265\cos 2\phi+0{\cdot}000162(t-\tau)]+\tfrac{3}{8}\left(6\frac{B}{760}-\pi\right)=\beta \tag{10}$$

we find from the equations (8) and (9)

$$\frac{\rho}{\rho_0} = \frac{\beta}{760}\frac{1-0{\cdot}000162t}{1+0{\cdot}003663t} \tag{11}$$

To form a serviceable table for the calculation of refraction we can now compute the quotient $\frac{\rho}{\rho_0}$ for a series of equidistant values of β and t, and then from (4) [by the help of $\frac{\rho}{\rho_0}$ and $\alpha_0 = 60''{\cdot}15 \sin 1''$ corresponding to ρ_0] we have the values of α; and when these are found the refractions corresponding to the adopted series of values of β and t can be calculated by (2). In order, therefore, to draw from the table thus obtained the refraction corresponding to the observed values of B, t, τ and π, there is the correction

$$\tfrac{3}{8}\left(6\frac{B}{760}-\pi\right)$$

to be applied to the indicated barometrical height B, besides the corrections dependent on h, ϕ, and $t-\tau$. The influence which this class of corrections has on refraction may easily be calculated by the aid of M. Radau's tables. The table marked I. by M. Radau indicates the mean refraction; Table II. gives, together with the arguments, zenith distance, and temperature, the variation of the mean refraction for 1° C., this variation—supposing that it is always taken as negative—is to be multiplied by t and added to the mean refraction. From Table IV., with the argument, mean refraction + correction for temperature, we can at length derive the variation of the refraction which corresponds to the change of 1^{mm} of the barometer. If this variation be multiplied by $\tfrac{3}{8}\left(6\frac{B}{760}-\pi\right)$ the required influence of the vapour pressure on refraction is the result.

I now give a list of the monthly means of π and t, which has been compiled from the observations of Professor Bauschinger in Munich (1891–93) and of Dr. Grossmann * in Vienna-Ottakring (1896–98):

	Munich. π mm.	Munich. t °	Ottakring. π mm.	Ottakring. t °
January	1·8	−12	...	
February	4·1	+ 1	3·9	+ 1
March	4·1	+ 3	4·2	+ 6
April	4·3	+ 7	5·7	+ 9
May	8·2	+14	8·9	+14
June	9·6	+14	10·4	+19
July	10·5	+17	...	
August	9·1	+14	12·7	+19
September	9·5	+12	10·4	+16
October	7·0	+ 7	7·1	+ 9
November	5·0	+ 1	4·0	+ 2
December	3·7	− 2	3·9	0

* "Beobachtungen am Repsoldschen Meridiankreise der von Kuffnerschen Sternwarte in den Jahren 1896–98," *Sitzungsberichte der königl. Sächsischen Gesellschaft der Wissenschaften*, Band xxvii. No. 1.

If we write $\frac{3}{8}(6-\pi)$ instead of $\frac{3}{8}\left(6\ \frac{B}{760}-\pi\right)$, which, however, is not strictly correct, we get for the influence of vapour pressure on refraction, at the zenith distances 55°, 65°, 70°, 75°, the following values :

	Munich.				Vienna-Ottakring.			
	55°.	65°.	70°.	75°.	55°.	65°.	70°.	75°.
	″	″	″	″	″	″	″	″
January	+0·19	+0·29	+0·37	+0·50	...	...	...	...
February	+0·08	+0·12	+0·15	+0·20	+0·09	+0·14	+0·18	+0·23
March	+0·08	+0·12	+0·15	+0·20	+0·08	+0·12	+0·15	+0·20
April	+0·07	+0·10	+0·13	+0·17	+0·01	+0·02	+0·02	+0·03
May	−0·09	−0·13	−0·16	−0·22	−0·11	−0·18	−0·23	−0·31
June	−0·14	−0·21	−0·26	−0·36	−0·18	−0·26	−0·32	−0·43
July	−0·18	−0·27	−0·34	−0·46	...	...	...	...
August	−0·13	−0·19	−0·24	−0·34	−0·27	−0·40	−0·50	−0·67
September	−0·14	−0·21	−0·27	−0·36	−0·18	−0·26	−0·32	−0·45
October	−0·04	−0·07	−0·08	−0·11	−0·04	−0·06	−0·08	−0·11
November	+0·04	+0·07	+0·09	+0·12	+0·08	+0·12	+0·15	+0·20
December	+0·10	+0·15	+0·20	+0·26	+0·09	+0·14	+0·18	+0·23

The errors arising from the neglect of the vapour pressure are noticeable even in the mean zenith distances ; they have moreover for the two specified series of observations from November to April the opposite signs to those from May to October. The zenith distances, without the corrections for vapour pressure, are too small in winter and too great in summer. The previously calculated corrections of refraction on account of vapour pressure can be represented by $f+x\sin\odot+y\cos\odot$, where x and y are functions of the zenith-distances. By way of example we have for Munich and $z=75°$ (under the supposition that the corrections hold good for the middle of the month) the following formula : Correction $=-0''{\cdot}03-0''{\cdot}34\sin\odot+0''{\cdot}21\cos\odot$. Supposing, then, the influence of moisture to have been omitted in the computation of refraction, the declinations (derived from the zenith-distances) of stars observed south of the zenith and the declinations concluded from the observations of northern stars at their lower culminations require a positive correction if observed in winter and a negative correction if observed in summer ; and these statements must be reversed for stars in upper culmination observed north of the zenith.

Besides the influence of damp here considered, there is still another effect which indeed almost disappears at $z=75°$, but is noticeable at greater zenith-distances, and must be alluded to in this reference to M. Radau's memoir, *Essai sur les Réfractions Astronomiques*, pp. 16, 17. On the other hand we must not fail to mention that M. Radau on the strength of the experiments of

Fizeau and Jamin, comes to the conclusion that the equation (7) should be replaced by

$$\rho = p^{\frac{1-\frac{1}{8}\frac{\pi}{p}}{1+at}} \tag{7a}$$

At the present time there are only two papers * published, in which, by means of astronomical observations, an attempt has been made to decide the question whether the formula (7) or (7a) is to be employed for the density of the air. Even if both investigations lead to the result that the former formula is to be preferred, yet a renewed examination of the question on the basis of the materials collected at other observatories is very desirable. In the meantime the corrections calculated by the writer, adopting formula (7), may be of interest.

Vienna-Ottakring: 1905 *June* 15.

On the Formula connecting Diameters of Photographic Images with Stellar Magnitude. By H. H. Turner, D.Sc., F.R.S., Savilian Professor.

1. The measures of the Oxford portion of the Astrographic Catalogue being now completed, it is possible to formulate certain conclusions as to the behaviour of the different plates in portraying stellar images. Early in the history of the work it was decided to record the diameters of the stellar images as an indication of the magnitude, and this was done throughout. But discussion of the results was deferred until a large amount of material had been accumulated.

2. The diameter of each star disc was estimated in units of $0''{\cdot}3$ in both positions of the plate, and the mean of the two estimates has been set down. No great precision is claimed for these estimates, and they are affected by a number of circumstances such as the following:

(*a*) Personal habits of the different measurers in estimating the limits of the ill-defined disc.

(*b*) Elongation of images near the corners of the plate. The observer was instructed to take the mean of the two diameters.

(*c*) Differences in intensity of image for the faint stars. In place of an estimate of diameter we have, then, an estimate of faintness.

3. In addition to these difficulties of interpreting the actual record on the plate, there is a systematic change in the impression

* Bauschinger, "Untersuchungen über die astronomische Refraktion," *Annals of the Munich Observatory*, vol. iii.; Courvoisier, "Unters. üb. die astr. Refr.," *Publications of the Heidelberg Observatory*, vol. iii.

made by a star of given magnitude according to its distance from the centre of the field. This change was studied by an indirect method, by counting the number of stars photographed at different distances from the centre of the plate (see *Monthly Notices*, vol. lxii. p. 434). It was found that the star density was by no means greatest at the centre of the plate, but reached a maximum at about 33′ from the centre, and that the density was liable to vary between 40 and 100, equivalent to something like one stellar magnitude. Hence to interpret the records of diameter so as to get accurate measures of stellar magnitude for all stars all over the plate would require a number of investigations for which it has not been possible to find the time. The records can only be regarded as an approximate guide to the magnitude.

4. In adopting a formula to convert the diameters into approximate magnitudes we therefore look, first of all, for simplicity, since we cannot hope for any great accuracy. The first trial made was of a simple linear formula,

$$\text{Diameter} = 13(10{\cdot}8 - \text{magnitude}),$$

and the estimated diameters of all the Cambridge stars in zones $+26°$ to $+31°$ were compared with theoretical diameters computed from this formula.

5. It was found—

(*a*) That though the deviations from a linear relation were small over a considerable range of magnitudes, they clearly manifested themselves in the mean of a number of observations. The errors of the above formula for the three groups of stars into which the Cambridge stars in zone $+26°$ were divided were as follows:

TABLE I. Zone $+26°$.

Group.	Approx. Mean Mag. (Camb.)	Theoretical Diameter.	Observed Diameter.	O−C.
Brighter than 8·0	7·0	49·40	48·29	−1·11
From 8·0 to 8·9	8·5	29·90	30·56	+0·66
From 9·0 to 9·5	9·3	19·50	19·36	−0·14

(*b*) That some plates gave systematically small diameters as compared with the formula, others systematically large (the former being no doubt generally taken on poor nights and the latter on good, though the sensitiveness of the plates is also an important factor), but that the defect or excess was not constant for stars of different brightness, as it might have been if the relation between magnitude and diameter were sensibly linear.

6. Thus there were in zone $+26°$ twenty-eight plates with observed diameters from five to ten units in defect (equivalent to 0·4 to 0·8 magnitude), and twenty-five plates with diameters in excess to the same extent. The mean errors of formula were as below for these two sets:

TABLE II. Zone +26°.

Camb. Mean Mag.	Theoretical Diameter.	Error of Formula.	
		Defective Plates.	Excessive Plates.
7·0	49·4	−10·9	+12·4
8·5	29·9	− 7·7	+10·1
9·3	19·5	− 6·8	+ 6·2

The error of diameter seems to be partly proportional to the diameter itself, and thus to represent the "defective" plates we must write

$$\text{Diameter} = 11{\cdot}7\ (10{\cdot}4 - \text{magnitude})$$

instead of the normal

$$\text{Diameter} = 13{\cdot}0\ (10{\cdot}8 - \text{magnitude})\ ;$$

while for the "excessive" plates we have

$$\text{Diameter} = 16{\cdot}1\ (10{\cdot}9 - \text{magnitude}).$$

These formulæ, that is to say, give approximately the same residuals as the mean formula gives for the normal plates, as is shown in Table III.

TABLE III. Zone +26°.

Mean Mag. Camb.	Defective Plates			All Plates.			Excessive Plates.		
	Obs.	Calc.	O−C.	Obs.	Calc.	O−C.	Obs.	Calc.	O−C.
7·0	38·5	39·8	−1·3	48·3	49·4	−1·1	61·8	62·8	−1·0
8·5	22·2	22·2	0·0	30·6	29·9	+0·7	40·0	38·6	+1·4
9·3	12·7	12·9	−0·2	19·4	19·5	−0·1	25·7	25·8	−0·1

7. If the above formulæ were strictly applicable, no star fainter than magnitude 10·4 could be shown on the "defective" plates; and indeed if the smallest recognisable diameter be taken as 4, this would give 10·1 as the magnitude of the faintest stars on the plate. Similarly for the "normal" plates we should have $D = 4$ when $m = 10{\cdot}5$, and for the "excessive" plates the faintest stars would be about 10·65. Now from independent considerations it seems probable that these limiting magnitudes are too low, and that the formula cannot be extended to magnitude 11·0.

8. The first natural thought is that the curvature indicated by the three residuals gives us the clue. We have done wrong to take a linear formula, and should have allowed for a slight curvature. But examination shows that the curvature indicated by the residuals is in the *wrong direction*. If we assume a formula between D, the diameter, and m, the magnitude, such as

$$D = a + bm + cm^2$$

(which is, of course, a purely empirical formula merely adopted to

introduce a curvature), and find a, b, c so as to give the observed diameters at magnitudes 7·0, 8·5, 9·3 as above, we get

$$D = 7{\cdot}4 + 3{\cdot}7m - m^2$$

and d vanishes when $m = 10{\cdot}5$ instead of when $m = 10{\cdot}8$, as before. Indeed, without any calculation it is clear that if the residuals after magnitude 9·3 become increasingly *negative*, the observed diameters are too small. If the plates really show stars brighter than is indicated by these linear formulæ we must look for a curvature in the *opposite direction*.

9. We are led to consider the scale of visual magnitudes adopted for reference. They are those of the Cambridge Catalogue, and were assigned as follows :

> "The circle observer, standing near the pointer microscope, with the observing catalogue and transit clock before him, directs the telescope for the coming star, clamps it, and announces the magnitude of the star. . . . The transit observer . . . finally calls out to the circle observer his estimate of the magnitude of the star, and any remark that may be thought useful for future reference."—[P. (6) of Introduction to Cambridge A.G. Cat. $+25°$ to $+30°$.]

Hence, being based on the magnitudes of the B.D., they are probably affected, partially at any rate, by the known systematic errors of that catalogue. Now Argelander's 8·5 should be 8·6, and his 9·3 should be about 9·9 (see *Harvard Annals*, vol. xxiii. p. 184).

10. Let us suppose for a moment that the Cambridge magnitudes are affected in *exactly* the same way. Then we should have the following comparison between the normal plate and the formula :

TABLE IV. Zone $+26°$.

Camb. Mean Mag. of Group.	Real Mean Mag.	Observed Diam.	Diam. by Formula 13 (10·8 − mag.)	New O−C.	Former O−C.
7·0	7·0	48·3	49·4	−1·1	−1·1
8·5	8·6	30·6	28·6	+2·0	+0·7
9·3	9·9	19·4	11·7	+7·7	−0·1

An approximate linear formula would now be

$$\text{Diameter} = 10\,(11{\cdot}8 - \text{mag.})$$

which gives O−C residuals of $+0{\cdot}3$, $-1{\cdot}4$, $+0{\cdot}4$ for the three groups, indicating a curvature *in the opposite direction* from before.

11. This brief discussion at any rate serves to show how much depends upon the magnitudes assumed for the fainter stars. A systematic error such as that affecting Argelander's magnitudes from 9·0 to 9·5 may not only alter the apparent magnitudes of stars near magnitude 11 by at least a whole magnitude, but may

actually change the *curvature* of the relation between magnitude and diameter from one direction to the other. So far as we have gone the linear relation seems to be very near the truth within the limits concerning us, and if there is any curvature it would seem to be probably in the direction opposite to that first indicated.

12. The linear formula first mentioned and part of the discussion of plates in zone +26° based on it represent the preliminary work of some years ago. Recently, since the completion of the measures of all zones, the diameters of zone +31° were compared with the same formula, viz.

$$\text{Diameter} = 13\ (10{\cdot}8 - \text{magnitude}).$$

The circumstances had meanwhile changed in several essentials; the measurers were new, and the plates were believed to have increased in sensitiveness. The developer for zone +31° was always Eikonogen, but for more than half zone +26° Hydrokinone was used. The dome had been renewed with a larger opening which may have affected the plates in some way (*e.g.* the telescope was more easily shaken by wind, though there was less danger of cutting off a part of the object-glass by oversight). Possibly the observer was more skilled in guiding, and there was certainly greater reluctance to take photographs on poor nights. For all these reasons a change in the formula would not be surprising; and one satisfactory result of the comparison declared itself at once—there were no seriously defective plates, *i.e.* none with defect as much as 5.

13. The mean plate for zone +31° when referred to the original formula

$$D = 13\ (10{\cdot}8 - \text{mag.})$$

had an excess of +4; and the observed diameters for the three groups are given in the second column of Table V. In the next two columns are given the corrections to two formulæ L (26) and L (31), viz.

$$D = 10{\cdot}0\ (11{\cdot}8 - m) \quad \ldots \quad \ldots \quad \ldots \quad L\ (26)$$

which was found at the end of § 10 for the observations of zone +26° with residuals +0·3, −1·4, +0·4; and

$$D = 8{\cdot}6\ (12{\cdot}7 - m) \quad \ldots \quad \ldots \quad \ldots \quad L\ (31)$$

which is the best linear formula for zone +31°.

Table V. Zone +31°.

Mean Corrected Mag.	Observed Diam.	O−C. L (26).	O−C. L (31).	O−C. Zone +26.
7·0	49·4	+1·4	+0·4	+0·3
8·6	34·0	+2·0	−1·2	−1·4
9·9	24·4	+5·4	+0·3	+0·4

14. The residuals for the two zones suggest a curvature of about the same amount. Let us form an estimate of its amount by adopting a formula

$$O-C = a+b(m-8{\cdot}6)+c(m-8{\cdot}6)^2$$

Then the values

$$a = -1{\cdot}3 \qquad b = +0{\cdot}4 \qquad c = +0{\cdot}8$$

are found to satisfy both sets of residuals fairly well. But of course the value of c is not very well determined. Adopting the above values, the linear formulæ should be replaced by

$$D = 30{\cdot}7-9{\cdot}6(m-8{\cdot}6)+0{\cdot}8(m-8{\cdot}6)^2 \ldots \quad \ldots \quad c(26)$$

and $$D = 34{\cdot}0-8{\cdot}2(m-8{\cdot}6)+0{\cdot}8(m-8{\cdot}6)^2 \ldots \quad \ldots \quad c(31)$$

15. Now the expressions on the right are not perfect squares, but they are not very different from perfect squares. The coefficient of $(m-8{\cdot}6)^2$ should in the first case be 0·75 and in the second case 0·50. If they had been perfect squares we could have taken the square roots and reduced the formulæ to the form

$$\sqrt{D} = p-qm$$

or $$m = a-n\sqrt{D}$$

which is that adopted at Greenwich ; and we conclude that this formula represents the Oxford results very accurately.

16. The reasons for adopting the formula

$$m = a-n\sqrt{D}$$

are given by the Astronomer Royal in *Monthly Notices R.A.S.*, vol. lii. pp. 125–146. The unit in which the Greenwich diameter is measured is 0″·15 (or half that adopted at Oxford) as mentioned on p. xxiii of the Introduction to vol. i. of the Greenwich Astrographic Catalogue. It is convenient to retain the symbol d for the diameter expressed in this way, and write D for diameter expressed as at Oxford in units of 0″·3 ; so that $d = 2D$. For six-minute exposures the mean formula found at Greenwich is

$$m = 13{\cdot}7-0{\cdot}77\sqrt{d}$$

or $$m = 13{\cdot}7-1{\cdot}08\sqrt{D}$$

The Oxford results give

For zone $+26^\circ$, $\quad m = 14{\cdot}9-1{\cdot}14\sqrt{D}$

„ $+31$, $\quad m = 16{\cdot}7-1{\cdot}39\sqrt{D}$

the errors of which formulæ are exhibited in Table VI. It will be remarked that we have now changed the independent variable from m to D.

TABLE VI.

Corrected Mean Mag.	Zone +26°. Observed $\sqrt{D}$.	Zone +26°. Calculated m.	Zone +26°. O−C.	Zone +31°. Observed $\sqrt{D}$.	Zone +31°. Calculated m.	Zone +31°. O−C.
7·0	6·95	6·98	−0·02	7·03	6·93	−0·07
8·6	5·53	8·60	0·00	5·83	8·60	0·00
9·9	4·41	9·87	−0·03	4·94	9·74	−0·06

17. The fact that the formula does not quite suit the Oxford results is thus exhibited in another way : the extreme residuals agree, but the middle one differs. The differences are not large when expressed in terms of magnitude, and a slightly different hypothesis as to the systematic errors of magnitude would smooth them out. We may proceed on the assumption that the law is sufficiently good for trial.

18. But attention is arrested by the difference in values of n. We should be prepared for a difference between Greenwich and Oxford, but there is an equal or greater difference between the two Oxford zones. What is this due to? We may obtain information by studying the valuable material given in the Greenwich Introduction.

19. Putting, then, the Oxford results aside for a time, we proceed to examine the formulæ on pp. xxviii–xxx of the Greenwich Introduction a little more closely to see if the variations exhibited in the constants a and n of the formula

$$m = a - n\sqrt{d}$$

are entirely accidental. It is readily seen that for a given exposure the constants a and n increase together. Collecting the results into groups they may be arranged as follows :

TABLE VII. (Greenwich).

Expos. 40m. No. of Plates.	Mean a.	Mean n.	Expos. 6m. No. of Plates.	Mean a.	Mean n.	Expos. 3m. No. of Plates.	Mean a.	Mean n.	Expos. 20s. No. of Plates.	Mean a.	Mean n.
5	16·9	0·92	7	14·4	0·84	3	13·7	0·88	2*	12·4	0·85
9	16·1	0·85	8	13·4	0·75	1	13·0	0·70	...	...	...
5	15·2	0·75	6	12·4	0·63	3	12·2	0·66	6	11·1	0·75

20. The general result is that for all exposures when a changes by 1·0, n changes by 0·10 in the same direction. If we plot m and $\sqrt{d}$ as abscissa and ordinate, the formulæ represent a series of straight lines of different slope ; but since the equation to one may be converted into that of any other by adding or subtracting a multiple of $1{\cdot}0 - 0{\cdot}10\sqrt{d}$, which vanishes when $d = 100$, all the lines for a given exposure will pass through the same point given by this value of d. If we consider the portions of the lines on the side of this point corresponding

* There is a printer's error, which makes it impossible to assign one of the values of a.

to small discs and faint stars, we naturally regard those nights as superior in quality which show faint stars well—*i.e.* show a large value of $\sqrt{d}$ for a given value of m, and those as inferior when the faint stars have small discs or are lost. But, following the lines back to the point of intersection, we come to certain bright stars with discs the same in all cases ; and crossing to the other side of this point we see that stars brighter still have discs which *increase* in size on nights when faint stars are lost. One obvious suggestion is that the variations we are concerned with arise chiefly from *travelling of images* rather than from absorption of light or failure in sensitiveness of the film ; for from either of the last two causes all stars should be affected in the same direction, while with unsteady images it is easy to understand both the loss of faint stars and the increase in size of the diameters of bright ones.

21. Thus adopting (as is done on p. xxx of the Greenwich Introduction) the formula

$$m = 13{\cdot}7 - 0{\cdot}77\sqrt{d}$$

as the mean formula for six-minute exposures, we may apparently adapt this to a good or bad night by writing

$$m = 13{\cdot}7 - 0{\cdot}77\sqrt{d} + v(10 - \sqrt{d})$$

where v is a quantity depending on the night, which may be either positive or negative. The question arises whether we should not adopt as standard the *best* night instead of the average night, and thus write, say,

$$m = (16{\cdot}0 - 1{\cdot}00\sqrt{d}) - v(10 - \sqrt{d})$$

where v is always negative, the Greenwich results indicating that the figures in the first bracket represent the best results.

22. This, however, is perhaps a matter of convenience. A more important question is this : In the mean formulæ found for the different exposures, viz.

Table VIII. (Greenwich).

for	40^m	$m = 16{\cdot}1 - 0{\cdot}84\sqrt{d}$
for	6^m	$m = 13{\cdot}7 - 0{\cdot}77\sqrt{d}$
for	3^m	$m = 12{\cdot}9 - 0{\cdot}76\sqrt{d}$
for	20^s	$m = 11{\cdot}5 - 0{\cdot}79\sqrt{d}$

may we not consider the variations in the value of n as due to the fact that the plates considered were taken on different nights, and reduce them for comparison to the same value of n by adding a suitable multiple of $(10 - \sqrt{d})$? If we take the formulæ as they stand and consider that the first term is of the form

$$c + b \log T$$

where T is the time of exposure and $b = 2{\cdot}5$ (assuming Pogson's scale of magnitudes), then taking 6^m as the unit of time, we get the following values for c :

				c
40^m	$16{\cdot}1-2{\cdot}5 \log 6{\cdot}67$	$=16{\cdot}1-2{\cdot}1$	$=$	$14{\cdot}0$
6^m	$13{\cdot}7-2{\cdot}5 \log 1{\cdot}00$	$=13{\cdot}7-0{\cdot}0$	$=$	$13{\cdot}7$
3^m	$12{\cdot}9+2{\cdot}5 \log 2{\cdot}00$	$=12{\cdot}9+0{\cdot}8$	$=$	$13{\cdot}7$
20^s	$11{\cdot}5+2{\cdot}5 \log 18{\cdot}00$	$=11{\cdot}5+3{\cdot}1$	$=$	$14{\cdot}6$

23. But if we reduce the formulæ to the same value of n (say 0·80 which is nearly the mean value) the first terms are altered and the values of c come out as below :

			c
40^m	$15{\cdot}7-2{\cdot}1$	$=$	$13{\cdot}6$
6^m	$14{\cdot}0-0{\cdot}0$	$=$	$14{\cdot}0$
3^m	$13{\cdot}3+0{\cdot}8$	$=$	$14{\cdot}1$
20^s	$11{\cdot}6+3{\cdot}1$	$=$	$14{\cdot}7$

The values of c now progressively increase instead of changing irregularly as before ; and this supports the view that it is right to reduce the formulæ to the same value of n in this way. But the progressive increase indicates that the value of b in the formula

$$a = c + b \log T$$

is *not* 2·5, but more nearly 2·0. If we adopt the value 2·0 for b the values of c would be accordant, thus :

		c
40^m	$15{\cdot}7-1{\cdot}6$	$=$ $14{\cdot}1$
6^m	$14{\cdot}0-0{\cdot}0$	$=$ $14{\cdot}0$
3^m	$13{\cdot}3+0{\cdot}6$	$=$ $13{\cdot}9$
20^s	$11{\cdot}6+2{\cdot}5$	$=$ $14{\cdot}1$

Hence to obtain one magnitude fainter we must increase the exposure time in the ratio

$10^{0{\cdot}5}$ instead of the ratio $10^{0{\cdot}4}$

i.e. in the ratio 3·16 „ „ „ 2·51

24. Some such conclusion has been several times stated before, *i.e.* it has been declared that prolonging the exposure 2·5 times did not give another magnitude ; and the Greenwich results seem to prove it clearly. As the reduction to the same value of n may not be generally accepted, however, I have examined the point in a different manner.

25. There are three plates (Nos. 6200, 6202, 6204) on which four exposures of 40^m, 6^m, 3^m, and 20^s were given, from which it is possible to determine the relation between m and T under specially favourable conditions. Let us take, for instance, plate

6200. The relationships found between m and d are given in the volume as follows:

		$d=9$.	$d=100$.
T = 40^m	$m_1 = 14{\cdot}6 - 0{\cdot}77\sqrt{d} =$	12·3	6·9
T = 6^m	$m_2 = 12{\cdot}0 - 0{\cdot}59\sqrt{d} =$	10·2	6·1
T = 3^m	$m_3 = 11{\cdot}9 - 0{\cdot}66\sqrt{d} =$	9·9	5·3
T = 20^s	$m_4 = 10{\cdot}8 - 0{\cdot}68\sqrt{d} =$	8·8	4·0

From these we can find the magnitude of a star with a given diameter for each exposure, as exhibited in the last two columns for values of $d = 9$ and 100 respectively. If now we put

$$m = c + b \log T$$

and find c and b by least squares, we obtain for

Plate 6200, d = 9	$m = 10{\cdot}61 + 1{\cdot}65 \log T$
d = 100	$m = 5{\cdot}84 + 1{\cdot}43 \log T$
Plate 6202, d = 9	$m = 11{\cdot}63 + 2{\cdot}20 \log T$
d = 100	$m = 6{\cdot}50 + 1{\cdot}86 \log T$
Plate 6204, d = 9	$m = 10{\cdot}95 + 1{\cdot}89 \log T$
d = 100	$m = 5{\cdot}39 + 2{\cdot}11 \log T$
Mean value of b	1·86

26. Or without using the formulæ we may go direct to the recorded measures and pick out equal diameters for the several exposures; but the result is the same, the coefficient of log T comes out sensibly smaller than 2·5, and is certainly not *greater* than 2·0.

27. There is a possible reason for this discrepancy which has not, so far as I know, hitherto received much attention. Assuming the visual scale correct, the short exposures give practically results which are *too good*. Now are they helped by the long exposures on the same plate? We know that exposure to a faint diffused light helps the production of star images; and the diffused light may be either (1) the general light of the sky, or (2) the unfocussed light of the star itself, which extends to sensible distances round the central image.

28. Mr. King, of the Harvard College Observatory, told me some months ago that, according to some experiments made by him, it makes no difference whether the subsidiary exposure to faint light precedes or follows the main exposure. Hence there is no need to consider the order in which the exposures are made.

29. The measures and formulæ given in the Greenwich Introduction give some material for testing this point. There are three plates (6200, 6202, 6204) with exposures of 40^m in addition to others of 6^m, 3^m, and 20^s; and there are also plates with the short exposures alone (6160, 6168, 6178). If the long

exposure of 40^m affects the short exposures in the manner indicated we ought to get a different progression in the value of c in the two cases.

To test the point the mean formulæ given in the Greenwich Introduction were formed and compared as follows :

Table IX. (Greenwich).

	Plates 6200, 6202, 6204 (with exposure of 40^m). m	Plates 6160, 6168, 6178 (short exposures only). m
6^m	$12{\cdot}8-0{\cdot}67\sqrt{d}$	$12{\cdot}7-0{\cdot}66\sqrt{d}$
3^m	$12{\cdot}7-0{\cdot}73\sqrt{d}$	$12{\cdot}7-0{\cdot}72\sqrt{d}$
20^s	$11{\cdot}0-0{\cdot}75\sqrt{d}$	$11{\cdot}1-0{\cdot}76\sqrt{d}$

The accordance is so good that it seems improbable that the long exposure had any effect, and this avenue of escape from the conclusion that we must increase the exposure time in a ratio considerably greater than 2·51 seems closed.

30. Is it possible that the scale of visual magnitudes is wrong ? Why should not the photographic evidence that a light is doubled be as good as that obtained by visual methods, which are equally liable to pitfalls ? The method of equalising two lights by use of a Nicol prism is certainly liable to systematic error from internal reflexions, as was pointed out by Dr. Spitta many years ago (*Monthly Notices R.A.S.*, vol. l. p. 325). Unfortunately for this explanation, the errors he dealt with are in *the other direction.* He found that when light was increased in a known ratio of 8 to 1 the erroneously evaluated wedge gave a *greater* ratio of 16 to 1 ; we find that when the light is increased in a known ratio by photographic exposure we get too *small* an apparent effect on the visual scale. This loophole of escape seems also closed.

31. I must not omit to refer to the paper by the Astronomer Royal (*Monthly Notices R.A.S.*, vol. lii pp. 125–146) in which he arrived at a conclusion at variance with that reached above—viz. that the value of b was sensibly 2·5, and not 2·0, or that to obtain one magnitude the exposure must be prolonged 2·512 times, and not 3·1 times. The value he obtains for the prolongation is actually 2·675, which is greater than 2·512, but not so large as 3·1. It is the mean of six values, 3·00, 2·80, 2·89, 2·06, 2·87, and 2·43, and the Astronomer Royal himself remarks that there is something curious about the plates giving the small results 2·06 and 2·43, which obviously affect the mean considerably. The evidence cannot, however, be neglected, though it is doubtful whether it can stand against the much greater mass of evidence given in the Greenwich Introduction.

32. It is perhaps not necessary to dwell further on this point, for it was twelve years ago admitted by Sir W. Abney that the law was liable to fail (see *Monthly Notices R.A.S.*, vol. liv. p. 65). He did not give very definite indications how the liability to fail was to be estimated, and it would seem as though we must

depend simply upon trial. Is it possible that the failure is due to a preliminary exposure to faint light by the makers of the plates, which is known to increase their sensitiveness? The general effect of such a preliminary exposure has been already considered in § 27; the short exposures would show up too favourably compared with the long, as they are found actually to do.

33. Whatever the reason, we seem to find, by a purely empirical process, that for the Greenwich results we may adopt the following general formula:

$$m = c + 2{\cdot}0 \log \mathrm{T} - n\sqrt{d} - v(10 - \sqrt{d})$$

where c and n are absolute constants, when once it has been decided what conditions of steadiness to adopt as a standard, and v depends on the night. Thus if we adopt $n = 0{\cdot}77$, as adopted in the Greenwich volume, we find from any of the mean formulæ

$$c = 13{\cdot}7$$

within $0{\cdot}1$, and the general formula may be written

$$m = 13{\cdot}7 + 2{\cdot}0 \log \mathrm{T} - 0{\cdot}77\sqrt{d} - v(10 - \sqrt{d})$$

or

$$m = 13{\cdot}7 + 2{\cdot}0 \log \mathrm{T} - 1{\cdot}09\sqrt{\mathrm{D}} - v(10 - 1{\cdot}41\sqrt{\mathrm{D}})$$

where m is the magnitude, T is measured in units of 6^{m}, d in units of $0''{\cdot}15$, and D in units of $0''{\cdot}30$.

34. Returning now to the Oxford results, we found in § 16 two sensibly different formulæ:

$$m = 14{\cdot}9 - 1{\cdot}14\sqrt{\mathrm{D}} \text{ for zone } +26°$$

and

$$m = 16{\cdot}7 - 1{\cdot}39\sqrt{\mathrm{D}} \text{ for zone } +31°$$

The former result is in fair agreement with Greenwich, but the latter differs sensibly from both.

There must be some reason for this considerable difference. It may be explicable by any of several changes made between the epochs at which the plates were taken, as mentioned in § 12. To test the effect of some of these the plates were grouped as follows: In each case the values of a and n were found from the first and third groups, and the value of a found independently from the middle group is added as a general numerical check and an indication of the error of the formula. The order of the groups is that of "defect" or "excess," according to the original formula (see § 6).

TABLE X.

Zone +26°; Plates 0–800; Developer Hydrokinone; Old Dome.

Group.	No. of Plates.	Mags. 7·0 and 9·9.		Mag. 8·6.
		n.	*a.*	*a.*
I.	10	1·13	13·88	13·82
II.	10	1·04	13·83	13·86
III.	10	1·00	13·74	13·63
IV.	10	1·14	14·62	14·56
V.	10	1·16	14·86	15·04
VI	10	0·98	14·27	14·31
VII.	7	1·07	15·32	15·03
Mean ...	67	1·07	14·36	14·32

TABLE XI.

Zone +26°; Plates 800–1557; Developer Eikonogen; Old Dome.

I.	10	1·12	14·06	14·06
II.	10	1·19	14·53	14·44
III.	10	1·19	14·84	14·69
IV.	10	1·17	14·77	14·68
V.	10	1·25	15·30	15·43
VI.	10	1·26	15·38	15·57
VII.	15	1·13	14·96	14·95
VIII.	10	1·25	15·78	15·78
IX.	10	1·18	15·47	15·55
X.	10	1·21	15·98	16·19
XI.	10	1·34	16·93	17·24
Mean ...	115	1·21	15·27	15·33

TABLE XII.

Zone +29°; Plates 0–800; Hydrokinone; Old Dome.

I.	13	1·03	14·11	14·12
II.	13	1·08	14·71	14·83

TABLE XIII.

Zone +29°; Plates 800–1557; Eikonogen; Old Dome.

I.	10	0·99	13·59	13·49
II.	10	1·00	14·08	13·92
III.	10	1·10	14·57	14·66
IV.	10	0·92	13·81	13·72
V.	10	1·07	14·68	14·60
VI.	10	0·96	14·24	14·11
VII.	10	1·01	14·50	14·58
VIII.	13	0·97	14·60	14·66
Mean ...	83	1·00	14·26	14·22

TABLE XIV.

Zone +29°; Plates 1558 to 2360; Eikonogen; New Dome.

Group.	No. of Plates.	Mags. 7·0 and 9·9.		Mag. 8·6.
		n	*a*	*a*
I.	16	1·18	15·17	15·17
II.	16	1·22	15·74	15·83
III.	16	1·31	16·86	16·84

TABLE XV.

Zone +30°; Plates 800–1557; Eikonogen; Old Dome

I.	8	1·10	14·01	13·98
II.	8	1·11	14·60	14·63

TABLE XVI.

Zone +30°; Plates 1557–2360; Eikonogen; New Dome.

I.	10	1·23	15·27	15·32
II.	10	1·27	15·74	15·59
III.	10	1·18	15·27	15·27
IV.	10	1·14	15·11	15·14
V.	10	1·35	16·44	16·28
VI.	10	1·32	16·16	16·37
VII.	10	1·27	16·07	16·05
VIII.	10	1·27	16·17	16·30
IX.	10	1·10	15·38	15·13
X.	10	1·40	16·97	17·14
XI.	10	1·39	17·11	17·32
XII.	10	1·30	16·75	16·74
XIII.	10	1·21	16·30	16·32
XIV.	13	1·27	16·99	16 94
Mean ...	143	1·26	16·12	16·14

TABLE XVII.

Zone +31°; Plates 1557–2360; Developer Eikonogen; New Dome.

I.	22	1·38	16·12	16·01
II.	26	1·33	15·99	15·93
III.	30	1·41	16·64	16·59
IV.	31	1·38	16·69	16·65
V.	30	1·34	16·84	16·53
VI.	19	1·35	17·23	17·15
Mean ...	158	1·36	16·59	16·48

35. The first thing that strikes us is that in any particular table the value of n does not seem to change very markedly with the value of a; take, for instance, Table XVII.: a changes by

more than 1·0, and from analogy with Greenwich we should expect a progressive change of 0·10 in the coefficient of d or 0·14 in that of D (see § 20). We do not find it. But taking the mean results for each table and arranging them in order we get Table XVIII. :

TABLE XVIII.

Table.	No. of Plates.	n	a	$0{\cdot}41\ a-n$.
XIV.	83	1·00	14·26	1·00
XIII.	26	1·05	14·40	0·98
XI.	67	1·07	14·36	0·96
XVI.	16	1·10	14·30	0·91
XII.	115	1·21	15·27	0·96
XV.	48	1·24	15·92	1·00
XVII.	143	1·26	16·12	1·01
XVIII.	158	1·36	16·59	0·98

in the last column of which the value of $0{\cdot}141\ a-n$ is calculated, and is seen to be approximately constant. The question is therefore raised whether the variations shown by the Greenwich results can be attributed to accidental causes, or are not systematic in some way.

36. *Developer.*—The change from Hydrokinone to Eikonogen made little difference. Taking the two zones which have plates of both kinds :

TABLE XIX.

Zone.	Hydrokinone.			Eikonogen.		
	Table.	No. of Plates.	n	Table.	No. of Plates.	n
+26°	XI.	67	1·07	XII.	115	1·21
+29°	XIII.	26	1·06	XIV.	83	1·00

the change goes opposite ways in the two cases. Moreover, though the values of n for zone +31° (developer Eikonogen) are large, those for +30° with the same developer are small.

37. *New Dome.*—Some change seems to have taken place with the erection of the new dome. Possibly the wider shutter-opening may have affected the plates, or they may have come up to a different focal reading after the dismounting of the object-glass during the operations.

TABLE XX.

Zone.	Old Dome.			New Dome.		
	Table.	No. of Plates.	n	Table	No. of Plates.	n
+29°	XIV.	83	1·00	XV.	48	1·24
+30°	XVI.	16	1·10	XVII.	143	1·26

But there must be also some other cause of change to explain the difference between zone +30° and zone +31°. At this point attention was drawn to the possible influence of the particular

measurer. The measurement of the diameter of a disc is a somewhat arbitrary operation in which there is room for considerable differences of personal habit; and the fact that zone +29° was measured chiefly by Mr. T. J. Moore, and gave a small value of n, seemed to point to the real existence of such differences. The quantities under discussion are, after all, small. To take a specific instance: if the diameter of a 9·9 magnitude star be taken as fixed at 20·0, then if the diameter of a 7·0 magnitude star is measured as

45 we get $n = 1{\cdot}30$

51 we get $n = 1{\cdot}10$

The difference of 6 units = 1″·8 would be a large quantity in estimating the position of the centre, but is not large for estimations of diameter of an ill-defined disc.

38. To test the point the results for zone +31° were divided into three groups according to the measurer. Three persons measured nearly all the plates, viz. Mr. B. Gray (B.G.), Mr. E. A. Gray (E.A.G.), and Mr. F. H. Scragg (F.H.S.). The available plates were divided into groups of ten in order of R.A. as a general check on the results.

TABLE XXI.

Zone +31°. Measurer, B.G.

Group.	No. of Plates.	Mags. 7·0 and 9·9.		Mag. 8·6.
		n	a	a
I.	10	1·32	16·07	16·06
II	10	1·42	17·22	17·01
III.	10	1·41	16·90	16·81
IV.	10	1·43	16·88	16·94
V.	7	1·44	17·12	17·04
Mean ...	47	1·40	16·84	16·77

TABLE XXII.

Zone +31°. Measurer, E.A.G.

VI.	10	1·39	16·29	16·22
VII.	10	1·35	16·18	16·27
VIII.	10	1·38	16·30	16·25
IX.	10	1·35	16·56	16·54
X.	10	1·39	16·77	16·62
XI.	5	1·27	16·26	16·00
Mean ...	55	1·36	16·39	16·32

Table XXIII.

Zone +31°. Measurer, F.H.S.

Group.	No. of Plates.	Mags. 7·0 and 9·9.		Mag. 8·6.
		n	*a*	*a*
XII.	10	1·14	15·19	15·00
XIII.	10	1·38	16·88	16·77
XIV.	10	1·41	17·11	17·03
XV.	8	1·14	15·34	15·18
Mean ...	38	1·27	16·13	16·00

39. The value of n thus changes sensibly from one measurer to another. For B.G., and to a less extent for E.A.G., its value comes out large. Hence we can explain the large value of n for zone +31° as compared with zone +26° at least *partially*; for zone +26° was practically completed before B.G. and E.A.G. came to the observatory. But, on the other hand, we cannot attribute the whole of these differences to personality, particularly that between zone +30° ($n = 1{\cdot}24$, see § 49) and zone +31° ($n = 1{\cdot}36$, see § 46); for the measurers were practically the same in the two zones, viz.

	B.G.	E.A.G.	F.H.S.	Others.
Zone +30°, No. of Plates	47	42	35	36
Zone +31 „	47	55	38	20

40. Indeed it is very difficult to explain the difference between these two zones. The plates for both were practically all taken with the new dome, and the measurers were approximately the same. Are the visual magnitudes compared with the photographic measures systematically different? As already remarked, those for zone +31° are Argelander's unchanged, while those for zone +30° are somewhat modified by some Cambridge estimations, which are used for one half the plate. For Argelander we have adopted the correction +0·6 to magnitude 9·3; and if D and D′ be the measured diameters of stars of magnitude 7·0 and 9·3 on Argelander's scale we find from zone +31°

$$\sqrt{D} - \sqrt{D'} = (9{\cdot}9 - 7{\cdot}0)/n = 2{\cdot}9/1{\cdot}36 = 2{\cdot}13$$

If the different value of n is attributable to the fact that the correction to the 9·3 stars for zone +30° is not +0·6 but $+x$ then

$$2{\cdot}13 = (2{\cdot}3 + x)/1{\cdot}24$$

or

$$x = 0{\cdot}34$$

Since nearly half of the stars on plates in zone +30° are Leiden stars with Argelander's magnitudes unchanged, the correction for the remaining (Cambridge) stars must be nearly zero to make the general mean 0·34. It thus seems probable that only a part of the difference in the value of n can be explained by a systematic difference in the visual magnitudes.

41. While these differences were under consideration one possible source of difference was examined with a negative result, which is nevertheless worthy of record. It was suggested that the difference in value of n for zones $+26°$ and $+31°$ might possibly be due to a difference of altitude of the stars photographed, or a consequent difference of atmospheric absorption. If there be any such effect, it ought to be shown by comparing plates of the same zone taken at different hour-angles. In zone $+31°$ there are 15 plates taken with hour-angle greater than 3^h, *i.e.* at Z.D. about 40° instead of about 20° near the meridian. The mean value of n for these plates was found to be 1·36, sensibly accordant with the mean; and it is concluded that within the working limits there is no appreciable effect from this cause.

42. It may also be mentioned that the plates in zone $+26°$ were grouped according to R.A., but no sensible dependence upon R.A. could be traced.

43. To sum up the preceding paragraphs, it is concluded that the changes in the value of n in the formula

$$\text{mag.} = a - n\sqrt{\text{diameter}}$$

may be ascribed *partly* to

(A) Changes in measures of the plates.

(B) Some alteration made when the new dome was erected; wider shutter opening (?) or unintentional alteration of distance between O.G. and plate (?).

(C) Possible systematic differences between Cambridge and Leiden magnitudes. And that

(D) Change of developer,

(E) Change of altitude, and

(F) Right Ascension

have no appreciable effect.

44. As regards longer exposures than 6^m at Oxford, the material for discussion is not large as yet. In *Monthly Notices*, vol. lxiv. p. 440, there is a paper by Dr. Metcalf on the region of *R Cygni*, in which he compares measured diameters on a plate of one hour's exposure with Hagen's visual magnitudes. The results may be grouped as follows:

TABLE XXIV.

Region of R Cygni.

Hagen's Mag.	No. of Stars in Group.	Diam.	$\sqrt{D}$.	$m+1{\cdot}36\sqrt{D}$.	$m+1{\cdot}00\sqrt{D}$.
4·9	1	144	12·0	21·0	16·9
7·0	1	80	9·0	19·2	16·0
8·6	6	54	7·4	18·7	16·0
9·2	11	43	6·6	18·2	15·8
9·7	10	40	6·3	18·3	16·0
10·5	8	34	5·8	18·4	16·3
11·5	11	27	5·2	18·6	16·7
12·1	13	21	4·6	18·3	16·7

45. In the last two columns two different values of the coefficient of D are tried : the larger value 1·35 suits the groups of stars better, but the two individual bright stars are better suited by the smaller value. Owing to the fact that individual stars may be coloured (the first star is almost certainly coloured, for Hagen gives magnitude 4·9, while the Rousdon magnitude is 6·4) we cannot decide between the values of n by any numerical process. The mean formula found in § 46 for zone +31° was

$$m = 16{\cdot}5 - 1{\cdot}36\sqrt{D}$$

which for a 60^m exposure should become

$$m = 16{\cdot}5 + b \log 10 - 1{\cdot}36\sqrt{D}$$

If we equate this to the formula found on the supposition that $n = 1{\cdot}36$

viz. $m = 18{\cdot}4 \qquad -1{\cdot}36\sqrt{D}$

(omitting the two bright stars in finding the constants) we get

$$b = 1{\cdot}9$$

which is in sensible agreement with the Greenwich result. But again we must not attach much weight to a single plate.

46. In a paper on Nova *Persei* Mr. Bellamy has compared the measured diameters of stars round it with Hagen's magnitudes (*Monthly Notices*, vol. lxi. pp. 479–80). The results are fairly well suited (excluding the faintest stars) by the formula

$$m = 15{\cdot}2 - 1{\cdot}36\sqrt{D}$$

but the exposures were short and not available for a determination of the coefficient of log T.

47. When Nova *Geminorum* was discovered, three long exposure plates (100^m, 35^m, and 30^m) were taken of the region (see *Monthly Notices*, vol. lxiii. pp. 326, 512). The diameters of a number of stars were measured at the time, but the information was not given in detail. The diameters of corresponding stars in the three plates were recently re-measured by F.H.S. Those stars whose visual magnitudes are given by Hagen were grouped according to his estimations as follows :

TABLE XXV.

Nova Geminorum Plates.

Group.	No. of Stars.	Hagen Mag.	$\sqrt{\text{Diam.}}$			$m+1{\cdot}20\sqrt{D}$.			$m+1{\cdot}10\sqrt{D}$.		
			100^m.	35^m.	30^m.	100^m.	35^m.	30^m.	100^m.	35^m.	30^m.
I.	4	7·7	8·5	7·4	7·4	17·9	16·6	16 6	17·1	15·8	15·8
II.	5	8·3	8·3	7·2	7·1	18·3	16·9	16·8	17·4	16·2	16·1
III.	6	8·9	7·2	6·4	6·3	17·5	16·6	16·5	16·8	15·9	15·8
IV.	6	10·3	6·6	5·2	5·0	18·2	16·5	16·3	17·6	16·0	15·8
V.	7	11·0	6·0	4·7	4·5	18·2	16·6	16·4	17·6	16·2	16·0
VI.	13	11·4	5·4	4·0	3·7	17·9	16·2	15·8	17·3	15·8	15·5
VII.	7	11·7	5·0	3·6	3·3	17·7	16·0	15·7	17·2	15·7	15·3
					Mean...	18·0	16·5	16·3	17·3	15·9	15·7

48. In the last six columns are given the inferred values of the constant a in the formula

$$m = a - n\sqrt{D}$$

on the two assumptions $n = 1{\cdot}20$ and $n = 1{\cdot}10$. The former seems to be about as large a value of n as we can admit; and the smaller value would suit the last three groups better, especially in the 30^m and 35^m exposures. But according to the Harvard determinations, Hagen's magnitudes of faint stars should be numerically increased, which may explain the apparent falling off. It is difficult to believe that the value of n should be smaller than 1·0; for the plates should be comparable with those of zone +31°, being recently taken; and although the measurer (F.H.S.) gets a smaller value of n than B.G. or E.A.G. his value found in § 38 is 1·27, though two groups fall as low as 1·14. On the two suppositions considered we may tabulate as corresponding formulæ:

m	log T.	$n = 1{\cdot}20$.	$n = 1{\cdot}10$.
6	0·00	$15{\cdot}6 - 1{\cdot}20\sqrt{D}$	$14{\cdot}9 - 1{\cdot}10\sqrt{D}$
30	0·70	16·3 ,,	15·7 ,,
35	0·76	16·5 ,,	15·9 ,,
100	1·22	18·0 ,,	17·3 ,,

The coefficient of log T got by comparing the 100^m and 6^m exposures is 2·4/1·22 or nearly 2·0; the 30^m and 35^m give a smaller value when compared with the 6^m exposure, and of course a larger value when compared with the 100^m.

Summary.

(A) In §§ 1–6 the measured diameters of star images on the plates of zone +26° taken at Oxford for the Astrographic Catalogue are compared with the linear formula

$$\text{Diameter} = p - q\ (\text{magnitude})$$

with which they nearly agree, the magnitudes being those of the Cambridge A.G. Catalogue, which are probably the same as those of Argelander. Residuals were formed for three main groups of magnitude 7·0, 8·5, 9·3 approximately, and the diameter of the middle group *exceeds* its value deduced from the extremes by about 1·3 unit = 0″·4.

(B) This excess is converted into a *defect* of 1·5 unit (= 0″·5) if we correct Argelander's magnitudes given above to 7·0, 8·6, 9·9 respectively; and the same holds for zone +31°, though the linear formula is different (§§ 7–13).

(C) The curvature is now approximately such as to agree with the form of relation adopted at Greenwich, viz.

$$\text{mag.} = a - n\sqrt{\text{Diam.}}$$

(D) But the values of n differ, not only Oxford from Greenwich, but zone $+26°$ from zone $+31°$. We turn first to the Greenwich published results to investigate such differences (§§ 16–18).

(E) At Greenwich, for a given exposure, a and n seem to vary together (§ 19). It is suggested that a variable term

$$v(10-\sqrt{d})$$

where d is measured in units of $0''{\cdot}15$ may be added or subtracted to the mean formula, according to the quality of the night.

(F) Reducing to the same value of n the formulæ given in the Greenwich Introduction for different exposures (40^m, 6^m, 3^m, 20^s), it is found that the constant a may be written

$$a = c + 2{\cdot}0 \log T$$

where c is independent of the time of exposure T. This corresponds to the following proposition: *When the time of exposure is prolonged in the ratio of five star magnitudes the photographic gain is four magnitudes* (§ 23).

(G) A few points are examined to see whether any explanation of this apparent loss can be found, but without success (§§ 27–33).

(H) Returning to the Oxford results, the plates in several zones are analysed to throw light on the changes in n. It is found (§§ 35–43) that n changes with the measurer, and with some alteration at the erection of the new dome ; but not with developer, altitude at exposure, R.A. There is also some unexplained change between zones $+30°$ and $+31°$.

(J) Although n does not change very much with a for plates taken under similar conditions, systematic changes in n, owing to change of conditions, follow changes in a, as in (E) above at Greenwich (§ 35).

(K) The coefficient $2{\cdot}0$ for log T, as in (F) above, is confirmed by such evidence as the scanty material for studying long exposures at Oxford offers (§§ 44–48).

University Observatory, Oxford :
1905 *May* 27.

Determinations of Stellar Parallax from Photographs made at the Cambridge Observatory. Introductory Paper. By Arthur R. Hinks, M.A. and Henry Norris Russell, Ph.D.

§ 1. Our purpose in the present paper is to give a general account of the investigations of stellar parallax undertaken at Cambridge with the Sheepshanks Equatorial in 1903. In that

year we planned a considerable scheme, which was put into operation forthwith owing to the appointment of one of us (H. N. R.) as a research assistant to the Carnegie Institution, and of which the first results are now available. The present moment seems therefore to be appropriate for an introductory account of the work. Two years' experience of the scheme in operation has shown us that it works well, and that the results come up to the standard of internal agreement which we set up. But since we hope that the work will be continued for some years, we should value present criticisms of the plan as matured by us, so that they may not come too late to enable us to avail ourselves of them.

§ 2. In making our plans we kept two main objects in view :

a. To secure the most advantageous relation between the amount of labour to be expended and the probable accuracy of the results to be achieved.

b. To eliminate at any cost all known sources of systematic error ;

And these led to the adoption of certain rules for the work, which may be briefly discussed as follows :

After careful consideration we decided not to adopt Professor Kapteyn's plan of accumulating exposures at successive epochs upon the same plate. It is well known that there are grave practical difficulties in carrying out this plan, which requires extreme care in the identification and preservation of undeveloped plates, great accuracy in adjusting the plate upon the telescope at each epoch, and, above all, reasonable certainty that the result of each separate exposure shall be successful, since the failure of one destroys the value of others as well. Further, we did not intend to limit the work on any object to three parallax epochs, but proposed to add a fourth and a fifth epoch for many stars, and to keep a few stars upon our working list for several years to serve as a control of the whole work. Again, considerations of another kind made it imperative to adopt the rule that all plates should be taken nearly on the meridian, which is a very severe restriction in itself in an uncertain climate, and renders it doubly undesirable that we should be hampered by any time-consuming processes. Finally, we did not want to be compelled to work for a whole year before any plates could be measured. We therefore decided :

I. *To take separate plates at each epoch and develope them at once* : Each plate has four exposures separated by 0·5 mm. in declination, and is impressed with a standard réseau. We intended to take two or three such plates at each epoch, but occasionally found it impossible to get more than one.

§ 3. Many of the stars which promised to give most interesting results were too bright to give measurable images on a plate exposed for three minutes, which is usually the minimum required to bring up enough comparison stars. We decided therefore to

use a special form of colour screen. Experiments showed that if one of the ordinary sensitive plates of patent plate glass was cleared, and a small patch of the gelatine film stained with yellow dye, a bright star photographed through this screen, held in the plate carrier almost in contact with the plate, could be diminished any desired number of magnitudes, and that definition was not sensibly affected anywhere in the field. It is easily shown that the effect of the parallel screen is to alter the focal setting by about one third of the thickness of the screen, while the scale value is unaffected. If the screen is 1·5 mm. thick and separated from the plate by a space of 0·5 mm., a deviation of the normal to the surface of half a minute of arc at the front, or one and a half minutes of arc at the back surface, produces a displacement in the image of 0·0001 mm. It follows that no great accuracy is required in figuring the surface of the screen, and it is probable that one could always find a piece of glass sufficiently good among one's spoiled plates.

The screen which we have used hitherto was made of worked glass by Messrs. Sanger Shepherd & Co., to whom we are much indebted for the interest they have taken in the experiment and the care which they have given to the work. A thin film of gelatine was coated on to the surface and dyed a deep orange ; it was then cut down to a patch 3 mm. square. This screen diminished the light of a star about six magnitudes. It was usually quite easy to set for and pick up the selected finding star so that the bright star fell well within the colour patch. When it was desired to photograph a star such as μ *Herculis* inside the patch with its ninth-magnitude companion about 30″ distant outside, the back of the plate carrier was taken off and the instrument set with the screen in place, the stars being viewed with a low-power eyepiece. When we were satisfied that this colour-screen method gave results which looked good, we added a number of bright stars to the programme and adopted the second rule :

II. *All stars brighter than* $6^{m}\cdot0$ *are to be photographed with a colour screen :* The only serious difficulty with these screens is that they are not permanent. We heard from Messrs. Sanger Shepherd that they had some difficulty in getting the small gelatine patch to adhere to the worked glass surface. They finally succeeded so well that after rather more than twelve months' use the patch, contracting, pulled off the face of the glass. Our series for bright stars were thus interrupted for the time. We realise that it is necessary to devise a form of screen which is permanent, not easily scratched, and which can be cleaned, and experiments are now in progress towards that end.

There is one objection which may be brought against the use of an orange screen to cut down a star's light : that the effective wave-length of the light from the bright star is not the same as that of the light from the comparison stars, and that conse-

quently atmospheric dispersion may produce an exaggerated effect. On the meridian this acts only in the y coordinate, which we do not generally use (see later). It should be remarked that the light which, after passing through the orange screen, forms the image of the star is not necessarily of greater effective wave-length than usual, since the orange dye may let through violet light. Should we adopt the orange dye for the final form of our screen, its absorption spectrum must be examined photographically.

§ 4. Let us now consider the various systematic errors which may be found in the plates, beginning with those of a permanent or semi-permanent kind.

The most serious are those included in the category of "hour-angle error," of which the two principal causes may be : (1) optical distortion varying with the hour-angle, especially when, as at Cambridge, a mirror forms part of the optical train ; (2) atmospheric dispersion. We realised from the outset that the way to ensure freedom from these errors was to work at a definite hour-angle for each star ; the only question was, Should we work always on the meridian, or choose in some cases an hour-circle not very far from it ? If we chose the meridian, and measured only the x coordinates, any effect of atmospheric dispersion must be eliminated ; but we should be sacrificing a good deal of parallax factor, and consequently of the weight of our determination.

It is not necessary to reproduce here the tables which were constructed to show what was the maximum parallactic displacement in x which could be obtained for any star, on the condition that it was photographed upon the meridian at a time when the Sun was at least 10° below the horizon. The whole available displacement in x is $1{\cdot}57\pi$ for a star in 0^h, rises to $1{\cdot}93\pi$ for a star in 6^h, falls to $1{\cdot}45\pi$ for a star in 16^h, and is then nearly constant as far as 22^h R.A.

The parallax factors for the limiting morning and evening observations under these conditions are usually unequal, and the total displacement might be somewhat increased by observing off the meridian in a particular hour-angle for each star. But there is a peculiar inconvenience about this procedure. The most favourable dates for observation of all stars between 15^h and 21^h R.A. are crowded into a few weeks ; and, what is worse, all these stars must be observed between $17^h\ 20^m$ and $18^h\ 40^m$ sidereal time. By departing from the meridian to gain a slight formal advantage in parallax factor we make a hopeless block in the observing programme. We have therefore adopted the simple working rule :

III. *All photographs must be taken within half an hour of the meridian :* It is very convenient to make the working list in the form of a card catalogue. Each star has a card on which the parallactic ellipse is drawn from the tables given by Sir David Gill (*Annals of the Cape Observatory*, vol. viii.). On the ellipse are

marked the places corresponding to the dates up to which evening observations and from which morning observations are possible. A glance at the card shows whether circumstances are favourable or unfavourable; whether the evening observations should be put off to the last moment, or may be made with equal advantage any time in the preceding month; and whether the morning observations must be got immediately after the earliest possible date, or may be delayed without damage. The conditions vary so much from star to star, especially in a latitude like that of Cambridge, which is really too high for convenient parallax work, that to have the diagrams always in sight is really necessary. They are made complete by a tracing from the B.D. chart to identify faint stars, and by the necessary miscellaneous instructions.

§ 5. Errors of the réseau, including "projection errors," scarcely enter into the parallax equations if reasonable care is taken in centering the plate: they affect only the star places.

An error of a semi-permanent kind is tilt of the plate, which displaces the centre of the plate with respect to the edges. It is particularly necessary to guard against such an error accumulating.

§ 6. Perhaps the most important error which is likely to affect all the exposures on one plate systematically, but to vary from plate to plate, is "guiding error." If the clock is not driving correctly, it is probably going pretty regularly either fast or slow, and the stars on the plate are continually trailing a little in one direction and being brought back: under these conditions a bright star image is displaced with reference to a faint by an amount which becomes quite sensible before the distortion of the disc is apparent on inspection. A good electric control will correct errors as soon as they amount to two or three tenths of a second of arc, and possibly the best visual guiding may do the same. The automatic control has this advantage, that one can set the clock regulator so that the accelerating and retarding trains come into operation with nearly equal frequency, which eliminates guiding error, properly so called. A slight continuous trail in one direction has little or no effect on the relative places of the images.

Another "plate error" is local distortion of the gelatine film, too local to be eliminated by the use of the réseau. Discordances appear from time to time which may be attributed with some confidence to this source. The remedy is to separate the different exposures well, and not to take too many on one plate.

§ 7. The plates are measured upon the Cambridge measuring machine (described in *Monthly Notices*, 1901, vol. lxi. p. 441), which has been found to be extremely convenient, accurate, and satisfactory. It has been found to be free from sensible errors of screws, scales, or optical distortion, with the exception of one very small term, which is rigorously eliminated when the plate is measured in two opposite orientations. The accidental error of

setting on a star-image appears to be very small, so that it does not pay to make many settings on a single image. We usually make two—more if the first two are discordant.

It is worth remarking in this connection that bad seeing does not usually appear to have much influence on the accuracy of the measures of a plate, though it may make the images diffuse.

At first all images were measured in two opposite orientations. It occurred to us later that if the error of measurement depending upon orientation was really systematic for images of a given intensity and appearance, it should be eliminated in the mean of the measures of several images of the same star by measuring half of them in the direct position only and the other half in the reversed.

Examination of the measures of about thirty plates showed that this was the case, and that very little accuracy would have been lost if the labour of measurement had thus been reduced by one half. We therefore adopted the rule :

IV. *Measure only two of the four exposures on a plate in the direct orientation, and the other two in the reversed :* § 8. It is possible to economise measurement still more. From the diagrams of our card-catalogue it appears that the available parallactic displacement for observations on the meridian is rarely more than half as great in the y coordinate (declination) as in the x, so that for parallax purposes the x's have on the average more than four times as much weight as the y's.

It is therefore desirable to confine exact measurements to the x's alone. To be sure we need approximate values of the y's in the reduction of the plates, but these can be obtained from rough measures of a single plate—in fact, of a single exposure—in perhaps $\frac{1}{20}$ of the time that it would take to measure the y's completely.

So we made the rule :

V. *Measure the* x*'s accurately on all plates, but the* y*'s approximately on one plate only :* This, again, saves half the work of measuring, and reduces it to a moderate proportion of the total work.

If the y's have weight enough to make it worth while to discuss them at all, this may be very easily done after the x's are finished, as will appear later.

§ 9. We have now to reduce our measures to a common standard system. Standard coordinates computed from catalogue places of stars are not well suited to our purpose. There are usually not enough catalogue stars on our plates ; the errors of their tabular places are much larger than those of the photographs ; and the necessary calculations take a good deal of time. The photographs themselves are free from all these objections. But a standard derived from them may differ slightly in scale value and orientation from the assumed values of these constants which we use in computing the parallax factors. These differ-

ences, however, are in practice so small that they cannot sensibly alter the deduced value of the parallax.

So our procedure is :

VI. *Choose any plate, or the mean of any number of plates, as a standard, and reduce the others to this :* Since all the necessary corrections for any given plate are linear functions of the coordinates it follows that the difference of the measured coordinates of the same star on two plates (barring accidental errors and proper motion) is a linear function of its coordinates on either one of the plates, or on any other plate of the same field, or even of the x of one plate and the y of another.

Each of our comparison stars (which as a first approximation we must assume to have no sensible parallax or proper motion) gives us then an equation of condition of the usual form

$$x - \xi = a\xi + b\eta + c$$

where x, y are the coordinates on the plate, and ξ, η the standard coordinates.

These equations might be discussed by least squares to find the plate constants a, b, c. But there is an approximate method, first proposed by Dyson, which has been found in practice to give about as good results with much less work. The following are some results of an investigation by one of us (H. N. R.) which will appear later.

(1) The least-square and approximate methods give identical values of the reduction to standard for a star situated at the centre of gravity of the comparison stars (considered as a system of equal particles in a plane). For other points the two methods give slightly different values, depending upon the differences of the constants a and b.

(2) When the comparison stars are distributed with tolerable symmetry in the four quarters of the plate, the values of a and b obtained by the approximate method differ from the least-square values by less than the probable errors of the latter.

(3) The weight of the reduction to standard determined by least squares is greatest for the centre of gravity, and falls off with increasing rapidity as we move away from this point in any direction. At the average (mean-square) distance of the comparison stars from the centre the weight is $\frac{1}{3}$ of its maximum value. If there are n comparison stars the maximum weight of the reduction is n times that of one equation of condition.

It follows from the above that the comparison stars should be so chosen that (*a*) their centre of gravity falls as near as possible to the parallax star, and (*b*) they are as evenly distributed as possible among the four quarters of the plate. If these conditions are fairly well satisfied, the approximate method of reduction may be used without loss of accuracy. Our experience shows that this can almost always be done in practice.

It should be remembered that in our equations of condition

we have expressed the correction necessary to reduce a plate to standard as a function of the standard coordinates. Consequently when our "parallax star" has a large proper motion, we must correct the standard coordinates of this star for this motion before using them to compute the quantity $a\xi+b\eta+c$. This correction is only sensible for a few very rapidly moving stars.

§ 10. If accidental errors alone had to be considered, it would pay us to use very few comparison stars, and to spend the time saved in measurement upon taking more plates. The error of a *reduced* coordinate of our parallax star consists of two parts : (*a*) the error of its own measured coordinate, and (*b*) that of the calculated correction to reduce it to standard. Only the latter part, which is the smaller of the two, can be diminished by using more comparison stars. Calculation on this basis shows that it would pay us best to use only four or five comparison stars, and take more plates, even if it took us twice as long to take a plate as to measure it.

But to take more plates, with our restrictions and climate, is not always possible, and there is an obvious objection to the use of so few comparison stars, for one of them may have a considerable proper motion or parallax much greater than the errors of observation.

This will alter the absolute term of one of our equations of condition by an amount compared with which the errors of the other equations may be neglected. If we had only three comparison stars, it would still be possible to find values of a, b, c which would exactly satisfy the three equations of condition. If we had four, consideration of a typical case shows that both the least-square and approximate solution give almost equal residuals for the four stars—two positive and two negative—so that we cannot pick out the bad one. Something of the same sort happens whenever the bad star is alone in the quarter of the plate in which it lies. To enable us to pick out possible cases of large proper motion or parallax among our comparison stars we must have at least two of them in each quarter of the plate.

In the great majority of fields it is possible to find eight measurable stars, such that their centre of gravity lies within a few minutes of arc of the parallax star, while there are two of them in each quarter of the plate.

It often helps us to satisfy the last condition if the lines dividing the plate into quarters (*i.e.* those separating the stars which are combined into groups in the approximate method of reduction) are inclined to the axes of coordinates.

§ 11. When all the plates of a field have been reduced, the residuals for the parallax star are converted into seconds of arc, reduced to a common epoch with the tabular proper motion of the star, and discussed by least squares in the ordinary fashion, the unknowns being corrections to the star's assumed x coordinate and proper motion, and its parallax. It is not worth while to

discuss the residuals for the comparison stars by least squares, because, as we have just seen, any parallax or proper motion in one of them will produce systematic changes in the residuals for all the others, so that the results for the different stars are not independent. Approximate values of the proper motions and parallaxes may, however, be very quickly obtained from the means of the residuals for each parallactic epoch. If any comparison star has a large parallax or proper motion the fact will appear by inspection of its residuals. It should then be abandoned as a comparison star, and put on the list of "parallax stars," and the plates be reduced anew with the remaining comparison stars. It is also worth while to measure approximately the y's on one plate of the last epoch, and reduced this to our standard, so that any large proper motion in declination may be detected.

In this way we may assure ourselves that none of our comparison stars has a large parallax. If we assume that the relative parallaxes and proper motions of these stars are really insensible, and that the values we have calculated for them are entirely due to accidental errors, we obtain a superior limit to the probable error of a parallax or proper motion derived from our plates. If the probable errors so obtained agree with those found from the least-square solution for the "parallax star" we may be satisfied that our comparison stars are really remote.

We may then pick out three of them, so that their centre of gravity falls near the "parallax star," measure accurately the y's for these four stars, reduce them to standard, and get from them a new determination of the parallax. This involves much less work than the discussion of the x's, for the reductions may be made very short. We are thus able to utilise the y's without too much work in proportion to their weight.

§ 12. Our solutions give us the difference between the parallax of our principal star and the mean of those of the comparison stars. We know the magnitudes of the latter and superior limits for their proper motions. Now Professor Kapteyn has shown that the mean parallax of a group of stars of known proper motion and magnitude can be predicted with considerable accuracy, so that for eight stars it is decidedly improbable that the actual mean will differ from the computed one by half the amount of the latter. For our comparison stars his formulæ give a mean parallax less than $0''.01$.

As we have already safeguarded ourselves against the exceptional case when one of our comparison stars has a large parallax, we may be satisfied that the uncertainty of their mean parallax will be small, and we may therefore pass from the relative to the absolute parallax of our principal star with some confidence when we desire to use the result for statistical purposes.

§ 13. There remain for consideration the principles that have guided us in forming our first working list.

It has been shown clearly by Kapteyn and Newcomb that the average parallax of stars is so small that if a large list of

stars is selected at random and observed, the spurious parallaxes will be far more numerous than the real parallaxes in the list of results: in other words, we are not yet in a position to attack the problem of parallaxes in general. While methods are still in an unsettled state, and while there is little evidence as to the real probable error of a parallax derived by photography, we shall do well to confine our attention to stars of two carefully selected classes.

A. Stars for which any information as to parallax, even if it be only a superior limit, is valuable—visual binaries, where the parallax gives the mass; variable stars, where the parallax gives the real range of light variation in terms of the light of the Sun; pairs of stars with common proper motion, where the parallax gives the size of the system; star clusters; and nebulæ.

B. Classes of stars which are likely to have larger parallax than the stars in general—bright stars and stars of large proper motion, especially the latter.

There will naturally be a wide difference of opinion as to the objects to be included under the first head; the second is in some respects easier to settle. We have tried to include the most promising stars of types I. and III., for which information is scanty. We have also included a number of stars of both classes which have been investigated elsewhere with discrepant results.

The observation of a great part of the objects in this first list is well advanced, and we hope to be able to publish first results for the whole within the next year or two. Our experience up to the present seems to show that one observer who can give his whole energy to the work may have with advantage about forty stars on his working list, of which he will miss a few at one epoch or another owing to spells of bad weather. In this country he will not accumulate very many more plates than he can measure to reduce if he imposes on himself the stringent rule of meridian observation which we have considered essential and adopts the simplified programme of measurement and reduction which we have used.

We should like to say in conclusion that we think it very desirable that those astronomers who are undertaking determinations of stellar parallax should follow the example of the line-of-sight spectroscopists and observe regularly a few stars in common. Only by some such informal co-operation does it seem possible to test what is the absolute accuracy of the work. We should therefore welcome suggestions for additions to our working list as well as criticism on the methods we have adopted.

First Working List for Stellar Parallax. Cambridge Observatory.

Name.	Position 1900·0. α h m	δ °	Mag.	Sp. Type D.C.	P.M. on Great Circle.	Notes.
ssiopeiæ ...	0 3·8	+58 36	2·42	F.	0″·55	0″·16 Pritchard; 0″·10 Flint
mbridge 34 ...	0 12·6	+43 27	7·9		2·80	0·29 Auwers; 0·31 Flint.
ssiopeiæ ...	0 42·9	+57 18	3·64	F.	1·20	Binary; 7^m comp. dist. 5′ (1900). 0·18 Peter; 0·44 Davis; 0·34 Flint.
er 20 ...	0 43·1	+ 4 46	5·7	H.?	1·37	0·16 and 0·12 Flint.
ssiopeiæ ...	1 1·6	+54 26	5·21	H.	3·75	0·13 Peter; 0·24 Bauer; 0·28 Jacoby; 0·07 Flint.
ssiopeiæ ...	1 5·0	+54 37	5	A.	0·22	0·23 Jacoby. In same field with μ.
ti	2 14·3	− 3 26	Var.	M.	0·24	
rsei	2 58·8	+38 27	Var.	M.	0·18	Irregularly periodic.
rsei	3 1·7	+40 34	Var.	A.	0·03	Sp. binary. 0·07 Chandler; 0·04 Chase.
nde 6888, 9 ...	3 40·2	+41 9	9; 9·5		1·38	Double; dist. 9″; −0·14 Flint.
turi	3 55·1	+12 12	Var.	B.	0·02	Algol var. 3·4 to 4·2; sp. binary, both comp. bright.
nde 7443 ...	3 56·5	+35 2	8·5		2·19	−0·02 Flint.
, 9012 ...	4 44·4	+45 41	7·5	A.	0·68	
minorum ...	6 8·8	+22 32	Var.	M.	0·07	Var. 3·2 to 4·2. Vis. and sp. binary.
minorum ...	6 58·2	+20 43	Var.	H.?	0·02	Var. 3·7 to 4·5. Sp. binary; probably triple.
minorum ...	7 28·2	+32 6	1·56	A.	0·21	Binary; both comp. sp. binaries. +0·20 Johnson; −0·17 Flint.
rdræ	8 41·5	+ 6 47	3·5	F.	0·21	Vis. and spect. binary.
sæ Maj. ...	8 52·4	+48 56	3·17	A.	0·50	0·13 Peters; 0·11 Flint.
Ursæ Maj. ...	8 54·2	+42 11	4·19	F.	0·51	0·20 Belopolsky; −0·07 Flint. Common P.M. with ι Ursæ.
mbridge 1646	10 21·9	+49 19	6·5	A.?	0·84	0·11 Kapteyn.
nde 21185 ...	10 51·9	+36 38	6·8		4·75	0·50 Winnecke; 0·43 Kapteyn; 0·37 Flint.
, 21258 ...	11 0·5	+44 2	8·5		4·40	0·26 Auwers; 0·26 Krüger; 0·17 Kapteyn; 0·37 Flint.
sæ Maj. ...	11 12·9	+32 6	4; 5	G.	0·74	Vis. binary; brighter comp. sp. binary.
e. 11677 ...	11 14·8	+66 23	9·0		3·04	0·25 Geelmuyden; 0·19 Bergstrand. Other stars near suspected of parallax.

Name.	Position 1900·0. α h m	δ ° ′	Mag.	Sp. Type D.O.	P.M. on Great Circle.	Notes.
					″	″ ″
83_1 Leonis ..	11 21·7	+ 3 33	6·5	I.	0·75	
83_2			7		0·68	
β Leonis	11 44·0	+15 8	2·07	A.	0·54	0·05 and 0·01 Pritchard.
Groombridge 1830	11 47·2	+38 26	6·5	A.?	7·05	0·18 Schlüter; 0·16 Kapteyn —0·01 Flint.
Lalande 22901 ...	12 7·8	+10 36	7·5		0·44	These three stars seem t form a group. The P.M. i almost entirely in dec southward.
„ 22908 ...	12 8·2	+11 24	7·5		0·59	
„ 22914 ...	12 8·4	+10 36	7		0·30	
γ Virginis .	12 36·6	− 0 54	3·0; 3·2	F.	0·58	Binary. Sp. Type I (McClean).
δ Virginis ...	12 50·6	+ 3 56	3	M.	0·51	
Lalande 25334 ...	13 34·7	+11 15	5·6	A.		0·25 Flint.
A.G.Berlin A.4999	13 40·2	+18 20	9·2		2·0	
Lalande 25372 ...	13 40·7	+15 20	8·5		2·32	0·43 Flint.
ξ Boötis	14 46·8	+19 31	4·5; 6·5	G.	0·16	Binary.
A. Oe. 14318 ...	15 4·7	−15 59	9·3		3·74	A pair with a remarkab common P.M.
„ 14320 ...	15 4·7	−15 54	9·2		3·74	
Lalande 27742	15 8·3	+19 39	7·5		0·67	A pair with proper motio essentially the same.
„ 27743	Δα + 0ˢ·31	Δδ + 23″·3	8·0		0·62	
OΣ 298	15 32·4	+40 9	7; 7·4		0·50	Visual binary.
$Weisse_2$ 720 ...	15 32·5	+40 8	7		0·50	Follows proceeding pa $5\frac{1}{2}^s$, 1′ 43″ N. The tw pairs have common P.M.
Lalande 29381 ...	16 1·5	+39 26	7		0·56	In the same field, but P.M in different directions.
„ 29439 ...	16 2·9	+38 55	8·5		0·60	
ζ Herculis ...	16 37·6	+31 47	3	G.	0·61	Binary, 3 and 6·5; mass equal. 0·15 Lewis (spe troscopic).
η Herculis ...	16 39·5	+39 7	3·69	K.?	0·08	0·40 Belopolsky; 0·15 Flin
W. B. XVII. 322 .	17 20·8	+ 2 14	8·0		1·36	0·17 Flint.
μ_1 Herculis ...	17 42·6	+27 47	4·0	I.?	0·81	A faint companion with cor mon P.M. is a binar 9·4 and 10.
Σ 2398	18 41·7	+59 29	8·2		2·27	Double; 17″; common P.M 0·35 Lamp; 0·32 Flint.
Munich I. 18180...	18 53·1	+ 5 48	9		1·26	
6 Cygni	19 9·5	+49 40	6·6	H.	0·64	Double; 10″. 0·48 Bal —0·02 Hall.
B.D. +30° 3639...	19 30·9	+30 18	9			Wolf-Rayet star with hydr gen atmosphere 5″ diameter.
Lalande 37647 ...	19 41·8	+33 22	8·5		0·47	Double.
„ 37686 ...	19 42·6	+33 34	5·5		0·47	Common P.M. with last.

Name.	Position 1900·0. α h m	δ ° ′	Mag.	Sp. Type D.C.	P.M. on Great Circle ″	Notes.
γgni	21 2·4	+38 15	5·11	H.	5″·16	0″·36 Wilsing, with suspected periodic motion of 61_1; Davis finds parallax of two stars sensibly different.
ulei... ...	21 9·6	+ 9 37	4·5	F.	0·30	Short period binary. 0·02 Flint; 0·02 Leavenworth; 0·07 Hussey (spectroscopic). Necessary to take plates through period of six years to eliminate effect of close binary.
ni	21 10·8	+37 36	4; 10	F.	0·48	Binary. 0·08 Beloposky.
de 43492 ...	22 12·3	+12 24	7	A.?	0·83	
o	22 24·5	+57 12	9; 11		0·95	Binary. 3″.
hei	22 25·5	+57 54	Var.	F.?	0·01	Sp. binary.
asi	22 58·9	+27 32	Var.	M.?	0·22	Irregular.
de 45755 ...	23 16·8	+48 33	7·5	A.	0·68	
dromedæ ...	23 32·7	+45 55	4	K.	0·45	Sp. binary.
de 46650 ...	23 44·0	+ 1 52	8·7		1·4	0·23 Flint.

Cambridge Observatory:
1905 *June* 2.

The Parallax of Lalande 21185 *and* γ *Virginis from Photographs taken at the Cambridge Observatory.* By Henry Norris Russell, Ph.D.

§ 1. The work upon which the writer has been engaged for the past two years as a research assistant of the Carnegie Institution has now progressed far enough to permit the publication of its first results. An outline of the methods employed, with the reasons which led to their adoption, is given in the preceding paper. The present communication deals with the numerical data obtained for the first two stars whose discussion has been completed.

§ 2. *Lalande* 21185.—

R.A. $10^h\ 57^m\cdot9$, Dec. 36° 37′ N. (1900·0), Mag. 7·3, P.M. 4″·77.

Previous investigations have shown that this is one of the nearest stars in the northern hemisphere, but they differ among themselves sufficiently to justify a fresh determination of its parallax.

The present discussion is based upon eight plates taken with

the Sheepshanks telescope (the first five by the writer, and the rest by Mr. Hinks), the circumstances being as follows :

No. of Plate.	Date.	Sid. T. h m	Exposures.	No. of Plate	Date.	Sid. T. h m	Exposures.
191	1903 Dec. 9	11 10	4	268	1904 Apr. 25	10 57	4
194	13	11 11	4	397	Dec. 30	11 6	3
258	1904 Apr. 16	11 3	4	405	1905 Jan. 9	10 43	4
260	19	11 10	4	426	Apr. 15	10 56	4

The fourth column gives the number of measurable exposures on each plate, and the third the mean of the times of the middle of these exposures.

The plates are coated on "patent plate" glass, and are of the size used for the astrographic chart, but owing to the longer focal length of the Cambridge telescope the field is a little less than $1\frac{1}{2}^{\circ}$ square. A standard Gautier réseau is impressed on all plates. The réseau interval of 5 mm. corresponds to $175''{\cdot}8$.

§ 3. There is a marked absence of stars in the N.E. part of this field, so that it was not possible to secure a perfectly symmetrical distribution of the comparison stars.

The following table shows the stars finally chosen, their B.D. numbers and magnitudes, the magnitudes given in the A.G. Catalogue (Lund) when they appear therein, and the approximate coordinates of the stars upon our plates the plate, centre being (20, 20). A denotes the "parallax star," Lal. 21185.

Star.	B.D.	Magnitude. B.D.	Lund.	*x*.	*y*.
1	+37 2142	8·3	8·2	9·87	29·37
2	36 2141	8·6	8·5	10·95	19·63
3	37 2145	6·8	7·5	13·04	32·13
4	37 2151	7·7	8·2	16·65	25·18
5	36 2144	9·1	—	17·25	11·87
6	36 2146	8·5	8·5	17·91	11·69
7	36 2150	8·9	8·8	25·99	14·56
8	36 2151	8·8	8·9	26·09	9·87
9	37 2153	8·5	8·4	32·70	32·54
A	36 2147	7·3	7·3	19·85	20·29
Centre of gravity of comparison stars				18·94	20·76

The centre of gravity falls very near the parallax star, but there is only one comparison star (No. 9) in the north-east quarter of the plate.

§ 4. On the first two plates all images were measured in both orientations, but on the others the first two were measured in the direct position and the last two in the reversed. The measures of individual images are carried to four decimal places (in terms

of a réseau interval), the last place corresponding to estimated tenths of a division of the micrometer head. The means of the coordinates of the four images of each star are then taken, and carried to five decimal places to avoid errors of computation. The differences from this mean are then tabulated for each exposure.

The scale value for the four exposures must be sensibly the same ; but the orientation may differ a little, owing to refraction and possible maladjustment of the polar axis of the telescope, and the centering for each exposure is of course different. If there were no accidental errors the differences from the mean should therefore be of the form $\triangle x = by + c$. The deviations from such a formula (which are easily obtained graphically) give a measure of the accuracy of the plate (though they will not show such things as "guiding error," which differs from star to star, but not from exposure to exposure). They also serve as a control of the numerical work, and to detect any errors that may have been made in recording the measures.

The y-coordinates were measured to three decimal places on one plate of each epoch.

§ 5. For the standard coordinates there were chosen the mean of the x's of Plates 191 and 194, with the y's of Plate 191. The approximate method of reduction may safely be applied in this case. It may be worth while to give an example of the method, say the case of Plate 258. Each comparison star gives us one equation of condition of the form

$$a\xi + b\eta + c = x - \xi$$

Taking the mean of the three equations in which ξ is greater than its mean value, and of the six in which it is less, we obtain

$$28{\cdot}262a + 18{\cdot}993b + c = +12531$$
$$14{\cdot}277a + 21{\cdot}646b + c = +11066$$

where the absolute terms are expressed in units of the fifth place. Similarly, from the four equations in which y is greater than its mean value, and the five in which it is less, we obtain

$$18{\cdot}066a + 29{\cdot}806b + c = +\ 9316$$
$$19{\cdot}638a + 13{\cdot}526b + c = +13346$$

From these two pairs of equations we find by subtraction

$$13{\cdot}985a - \ 2{\cdot}653b = +1465$$
$$-\ 1{\cdot}572a + 16{\cdot}280b = -4030$$

whence $\quad a = +58{\cdot}88 \qquad b = -241{\cdot}84$

and from any one of the first four equations

$$c = +15460$$

By calculating c from all four of these equations we get a control for the numerical work.

We next calculate the residuals for each star in the sense $x-\xi-a\xi-b\eta-c$. The sum of the residuals for each set of comparison stars which was grouped together above must be zero. This gives a searching control on the reduction.

§ 6. The residuals for the parallax star are now converted into seconds of arc and reduced to the epoch 1904·0 with the star's proper motion taken from Bossert's catalogue, which in the present case is $-0^{s}\cdot044$ or $-0''\cdot53$.

We then obtain the following equations of condition, in which δx denotes the correction to the standard x-coordinate for 1904·0, $\delta\mu$ the correction to Bossert's proper motion in R.A., and π the parallax of our star relative to the mean of the comparison stars, while the absolute terms are given in thousandths of a second of arc.

				O−C.
1·000δx	−0·061$\delta\mu$	+0·907π	= − 18	+20
1·000	−0·051	+0·900	= − 41	− 1
1·000	+0·291	−0·634	= −538	+34
1·000	+0·299	−0·668	= −589	− 5
1·000	+0·315	−0·736	= −650	−49
1·000	+0·999	+0·817	= −139	−66
1·000	+1·026	+0·733	= − 58	+44
1·000	+1·288	−0·621	= −549	+23

The influence of the parallax is very conspicuous in the absolute terms.

Our normal equations are as follows :

8·000δx	+4·106$\delta\mu$	+0·698π	= −2588
4·106	+3·989	+0·051	= −1442
0·698	+0·051	+4·613	= +1349

whence

δx = −351·7	weight	3·68
$\delta\mu$ = − 4·0	,,	1·86
π = +345·7	,,	4·50

The residuals left on substituting these values in the equations of condition are given above under the heading O−C. The sum of these squares is 10,804, whence we derive

Probable error of one equation				...	...	±31·4
,,	,,	δx	,,	...	...	±16·5
,,	,,	$\delta\mu$	,,	...	...	±23·0
,,	,,	π	,,	...	...	±14·8

We have thus for the definitive result of the measures in x

$$\pi = +0''\cdot346 \pm 0''\cdot015$$

and for the probable error of one equation, *i.e.* of a coordinate derived from one plate, $\pm 0''\cdot031$.

§ 7. We may now investigate the parallaxes and proper motions of our comparison stars. In this case we are justified in an approximate but much shorter form of solution. If $\Delta_1 \ldots \Delta_8$ denote the absolute terms in the successive equations of condition for any star, we easily find by combining the equations in which the factors of π have the same sign

$$1\cdot000\delta x + 0\cdot548\delta\mu - 0\cdot665\pi = \tfrac{1}{4}(\Delta_3+\Delta_4+\Delta_5+\Delta_8)$$
$$1\cdot000\delta x + 0\cdot548\delta\mu + 0\cdot831\pi = 0\cdot217(\Delta_1+\Delta_2) + 0\cdot283(\Delta_6+\Delta_7)$$

whence we obtain by subtracting and then dividing by 1·496

$$\pi = +0\cdot145(\Delta_1+\Delta_2) + 0\cdot189(\Delta_6+\Delta_7) - 0\cdot167(\Delta_3+\Delta_4+\Delta_5+\Delta_8)$$

Similarly by constructing two equations in which the coefficients of δx and π are the same, but those of $\delta\mu$ widely different, we find

$$\delta\mu = 0\cdot325(\Delta_6+\Delta_7+\Delta_8) - 0\cdot306(\Delta_1+\Delta_2) - 0\cdot123(\Delta_3+\Delta_4+\Delta_5)$$

Applying these formulæ as a test to our parallax star, we find $\delta\mu = -5$, $\pi = +343$, in very good agreement with the least-square solution.

For our comparison stars we find in the same way, in thousandths of a second—

Star.	1.	2.	3.	4.	5.	6.	7.	8.	9.
μ	+21	+84	−26	−42	−9	−23	+ 9	−58	+48
π	−32	+18	0	− 5	+7	+ 7	−19	−18	+35

The sum of all the proper motions or of all the parallaxes vanishes, as it ought to do, since they are all relative to the mean of the group.

If we assume that these values are wholly spurious, and due to errors of observation, we find for the probable errors of a proper motion or parallax for one comparison star the values ± 30 and ± 14 respectively. Comparing these with the values for the parallax star we see that the values of π are completely accounted for by accidental errors (supposing these to be the same for the comparison stars and parallax star), while those of μ are a little larger than the accidental errors would lead us to expect. The large value for star 2 may perhaps be real.

If we assume that our comparison stars have no parallax or proper motion (or, rather, that they all have the *same*), the differences of the residuals on different plates will be due to errors of observation. In this way we obtain for the probable error of a coordinate derived from one plate values which

range from $\pm 0''.018$ to $\pm 0''.046$ for the different stars, the mean value being $\pm 0''.030$. As this has been derived from residuals left after the reduction of the plates to standard, in which we had to determine three unknowns from eight equations, we must multiply it by $\sqrt{\frac{8}{5}}$ in order to obtain a number comparable with the one previously found for the parallax star. We thus obtain $\pm 0''.038$ for the true probable error of an x-coordinate of a comparison star derived from one plate. This is somewhat larger than the value for the parallax star, perhaps because the comparison stars really have small proper motions of their own.

It is of interest to compare the agreement of the plates with one another with that of the different exposures on one plate, which can be found from the differences mentioned in § 4.

The average value (without regard to sign) of these discordances for all the stars measured on the eight plates is 3·12 units of the fourth place, or $0''.055$. To find the corresponding probable error of a single image we must multiply by the constant 0·845, and also by $\sqrt{\frac{4}{3}}$, since we are considering deviations from the mean of four quantities, and $\sqrt{\frac{10}{8}}$ because we have tried to represent ten quantities for each exposure by a formula with two constants. This gives for the probable error of one image $\pm 0''.060$. That of the mean of four images would then be $\pm 0''.030$, which is close to that found from the agreement of different plates. We may therefore conclude that for these plates the "plate errors" are very small.

The reduction of the approximate values of y for the four epochs gives residuals for the comparison stars that lie within the errors of the measures, showing that their proper motions in declination, like those in R.A., are all small.

§ 8. We pass now to the discussion of the y's. For this purpose three of the comparison stars were chosen—Nos. 2, 6, and 9—whose centre of gravity falls within one réseau-interval of the parallax star, and whose parallaxes all appear to be very small. The y's of these four stars were measured accurately on all the plates. The reduction to standard is in this case very simple. If ξ_2, η_2, denote the standard coordinates of star 2, and so on, we determine three auxiliary constants, α, β, γ, by the equations

$$\begin{aligned} \alpha\xi_2+\beta\xi_6+\gamma\xi_9 &= \xi_A \\ \alpha\eta_2+\beta\eta_6+\gamma\eta_9 &= \eta_A \\ \alpha+\beta+\gamma &= 1 \end{aligned}$$

Then if we denote any expression of the form $a\xi+b\eta+c$ by f, we will have

$$f_A = \alpha f_2+\beta f_6+\gamma f_9$$

The correction to reduce the place of the parallax star to standard may thus be derived immediately from the differences from standard for the three comparison stars.

The results obtained in this case are interesting as showing how conspicuous a large proper motion is, even on photographs taken at short intervals. In the table below the first line gives the residuals in thousandths of a second of arc; the second, the correction necessary to reduce them to 1904·0 with Bossert's proper motion, $-4''\cdot74$; and the third, the corrected values:

Plate ...	191	194	258	260	268	397	405	426
Residual...	+ 33	− 33	−1419	−1436	−1486	−4984	−5102	−6035
Correction	−289	−242	+1379	+1417	+1493	+4735	+4863	+6105
Corrected value	−256	−275	− 40	− 19	+ 7	− 249	− 239	+ 70

Our equations of condition are:

				O−C.
1·000δy	−0·061$\delta\mu$	−0·294π	= −256	+40
1·000	−0·051	−0·256	= −275	+ 9
1·000	+0·291	+0·595	= − 40	−41
1·000	+0·299	+0·588	= − 19	−18
1·000	+0·315	+0·571	= + 7	+13
1·000	+0·999	−0·078	= −249	−24
1·000	+1·026	+0·027	= −239	−49
1·000	+1·288	+0·596	= + 70	+70

The influence of the parallax is again conspicuous.

The normal equations are:

+8·000δy	+4·106$\delta\mu$	+1·749π	= −1001
+4·106	+3·989	+1·277	= − 389
+1·749	+1·277	+1·540	= + 169

Whence we find

	Weight.
$\delta y = -197\cdot8$	3·61
$\delta\mu = -\ 1\cdot5$	1·76
$\pi = +335\cdot5$	1·08

The residuals in the equations of condition are given above. From them we derive:

Probable error of δy	$\pm$17
$\delta\mu$	$\pm$25
π	$\pm$31
One equation	$\pm$33

The definition solution from the y's gives therefore

$$\pi = +0''\cdot335 \pm 0''\cdot031$$

The probable error of a y-coordinate derived from one plate is almost exactly the same as that of an x-coordinate, but the latter gives a determination of the parallax with four times as much weight as the former. The agreement of the two values is very satisfactory. Combining them with regard to these probable errors, we have for our final value, relative to the nine comparison stars—

Parallax of Lalande 21185 = 0″·344±0″·013

§ 9. The following table gives in summary form the result of previous investigations of this star's parallax :

Observer.	Date.	Method.	Number of Comp. Stars.	Number of Obs.	Result.
(1) Winnecke	... 1857–58	Heliometer	2	12	+0″·511 ±0″·015
(2) Kapteyn	... 1885–87	Transits	2	46–47	+0″·434 ±0″·028
(3) Flint ...	... 1893–95	Transits	2	18	+0″·36 ±0″·047
		or, including a systematic correction,			+0″·37

References: (1) *A.N.* 1147. (2) *A.N.* 2935. (3) *Publications of the Washburn Observatory*, vol. xi. pp. 219, 437.

The present investigation supports the most recent ones in showing that the parallax is smaller than at first supposed, so that this star is not the nearest in the heavens after *α Centauri*, but is more remote than *Sirius*, and probably 61 *Cygni* as well.

§ 10. We have still to consider the effect of atmospheric dispersion on our results. The displacement of a star-image on the plate by refraction is given by the equations

$$\Delta x = \beta \mathrm{X} + \text{small terms}$$
$$\Delta y = \beta \mathrm{Y} + \text{small terms}$$

where β is the constant of refraction, and X, Y the coordinates of the zenith projected on the plane of the plate, expressed in terms of the focal length as unit.

If the effective mean wave-length of the light of the parallax star differs from that of the comparison stars, the refraction constant will also differ, say by $d\beta$, and the parallax star will be displaced on the plate relatively to the others by $\mathrm{X}d\beta$ and $\mathrm{Y}d\beta$ in the two coordinates.

For plates taken near the meridian we have (neglecting terms involving the cube of the hour-angle)

$$\mathrm{X} = \frac{t\cos\phi}{\cos(\phi-\delta)},\quad \mathrm{Y} = \tan(\phi-\delta) + \tfrac{1}{4}t^2 \sin 2\phi \sec^2(\phi-\delta)$$

where ϕ is the observer's latitude, δ the declination of the plate centre, and t the hour-angle expressed in circular measure. The dispersion in x is therefore proportional to the hour-angle, and vanishes at the meridian, while that in y is practically constant for each field.

Computing thus the effect of refraction for each of our plates, and introducing the results into our equations of condition and normal equations, we find for the effect on our unknowns :

Measures in x.	Measures in y.
$d\delta x = +0\cdot028 d\beta$	$d\delta y = +0\cdot280 d\beta$
$d\delta\mu = -0\cdot034 d\beta$	$d\delta\mu = +0\cdot000 d\beta$
$d\pi = +0\cdot002 d\beta$	$d\pi = +0\cdot000 d\beta$

Here $d\beta$ denotes the change in the refraction constant expressed in seconds of arc. As the whole difference between the refraction constants for the visual and photographic rays is less than $1''$, it is clear that our results must be free from any sensible error arising from this source, except as regards δy, whose exact value is quite immaterial.

It should, however, be noticed that we have been regarding $d\beta$ as constant, whereas it really varies with the meteorological conditions proportionately to the total refraction. This cannot affect our x-equations, where the coefficients of $d\beta$ are all very small ; but as the change is a seasonal one it may produce some effect on the value of the parallax derived from the y's. The refraction averages greater in winter than in summer ; for our star Y is positive ; therefore the star will appear farther north in winter than in summer, if $d\beta$ is positive. But the effect of annual parallax is to displace a star to the southward in winter and northward in summer.

Consequently if $\delta\beta$ is positive—that is, if the star is bluer than the comparison stars—the effect of seasonal variations in the dispersion will be to make the value of the parallax found from the y's too small. This effect is, however, a small quantity of the second order, and is probably quite insensible.

§ 11. We have finally to consider what is the probable parallax of our comparison stars. We have already found that their relative proper motions and parallaxes are very small. The very small values of the corrections found to the catalogued motion of the parallax star, which is very well determined, show that our comparison stars have no common drift. Their proper motions as computed from our plates are probably largely due to accidental error. If we assume that the true motions and the errors of observation contribute equally to the observed results, the observed proper motions in one coordinate will on the average be equal to the true proper motions in the plane of reference.

We may then apply Professor Kapteyn's formulæ for the mean parallax of a group of stars of given proper motion and magnitude given in No. 8 of the Publications of the Astronomical Laboratory of Groningen. The average magnitude of our comparison stars is 8·3, and their average observed proper motion in x, without regard to sign, is $0''\cdot036$. With these arguments

Kapteyn's table [*loc. cit.* p. 31, Table G, headed "All the Stars"] gives mean parallax = 0″·0071.

If we discard all hypotheses concerning the proper motions and use the magnitude alone as the criterion of distance, Kapteyn's Table C [*loc. cit.* p. 28] gives mean parallax 0″·0074.

We may therefore assume with some confidence for our comparison stars

Mean Parallax = 0″·007

From Kapteyn's researches it appears that it is more likely than not that the parallax of a single star will be within 50 per cent. of the value given by his table for a star of its magnitude and proper motion. For the mean of nine stars we should have a much closer agreement, so that the value just found is not likely to be in error by more than a very few thousandths of a second, especially as we have already seen that none of the stars has a large parallax.

By adding this to the value already found for the parallax of *Lalande* 21185 relative to the comparison stars, we may obtain a very close approximation to its absolute parallax, and this should be used rather than the relative parallax in computing the star's distance, light, and the like.

§ 12. γ *Virginis* R.A. $12^h\ 36^m{\cdot}3$. Dec. 0° 55′ S. (1900·0). Binary. Components equal: joint magnitude 2·91. P.M. 0″·57. Pos. 327°. Dist. 5″·7 (1904).

This bright star was photographed through the colour screen, and eight plates at three epochs were secured before the failure of the latter.

Except on very unsteady nights the images of the two components are well separated; but to ensure this the exposures had to be short, and, as the field is a very poor one, it was found impossible to get the ordinary number of measurable comparison stars. If we had had a series of colour screens of varying densities this could have been remedied by using a denser screen and longer exposures; but, as things were, it was necessary to get along with only six comparison stars—the smallest number for any of our fields. It also appeared early in the course of measurement that these plates were below the average in quality, owing perhaps to the relatively low altitude of the star, which is one of the southernmost on our list. One of the plates was shown by the discordance of the four exposures to be particularly bad, and it was given half weight, a decision confirmed later by the large residuals which it gave in the final solutions.

The present discussion may therefore be taken as an example of our photographs at their worst, and it is gratifying to find that even then they give results of some apparent value.

The general plan of the work was exactly similar to that for the previous star, so that only the points of difference need be mentioned here.

§ 13. Having only six comparison stars the method of reduction was somewhat altered. The stars were divided into three pairs, and the means of the equations of condition for each pair were taken, thus giving three equations for the three plate constants. As the centre of gravity of the six stars fell within a réseau interval of the parallax stars, the use of this approximate method is justifiable.

Solutions were made for the two components separately, the parallaxes of the comparison stars were approximately determined, three of them were chosen and the y's measured, with the results given below. A denotes the southern and B the northern component of the binary, and the assumed proper motions are $-0^{s}\cdot038$ ($= -0''\cdot57$) in x and $+0''\cdot015$ in y.

From measures in x.

	Star A.	Star B.	Weight.
$\delta x =$	$-0''\cdot029 \pm 0''\cdot030$	$-0''\cdot019 \pm 0''\cdot038$	2·66
$\delta\mu =$	$+0''\cdot110 \pm 0''\cdot046$	$+0''\cdot089 \pm 0''\cdot059$	1·14
$\pi =$	$+0''\cdot072 \pm 0''\cdot027$	$+0''\cdot054 \pm 0''\cdot034$	3·34
Probable error of unit weight	$\pm 0''\cdot049$	$\pm 0''\cdot063$	

From measures in y.

$\delta y =$	$+0''\cdot028 \pm 0''\cdot037$	$+0''\cdot026 \pm 0''\cdot079$	2·64
$\delta\mu =$	$-0''\cdot088 \pm 0''\cdot057$	$-0''\cdot107 \pm 0''\cdot121$	1·12
$\pi =$	$+0''\cdot070 \pm 0''\cdot074$	$+0''\cdot068 \pm 0''\cdot157$	0·67
Probable error of unit weight	$\pm 0''\cdot061$	$\pm 0''\cdot128$	

The weight of the parallax derived from the y's is but one fifth of that from the x's (and even this is more than it would be for the average star). It would not ordinarily pay to measure them ; but as the present series cannot be continued, it seemed worth while to get all possible information out of the plates.

The large probable errors found for the y coordinates of star B are due to one very large residual for the plate which had previously, for quite other reasons, been given half-weight.

If we combine the results from the x's and y's with regard to their probable errors, we have

Parallax of A	$+0''\cdot072 \pm 0''\cdot024$
B	$+0''\cdot054 \pm 0''\cdot033$

The two values agree within their probable errors. Taking the mean with equal weights, we have for the parallax of *γ Virginis* relative to the six comparison stars

$$\pi = +0''\cdot063 \pm 0''\cdot022$$

There is, however, something unsatisfactory about this solution. The proper motion of γ *Virginis* (which is in the Fundamental Catalogue) is very well determined, and the large corrections found above are almost certainly not real. It is indeed barely possible that the comparison stars have a "group motion" which accounts for the discrepancy; but this is exceedingly improbable, and the large probable errors of the calculated values of $\delta\mu$ suggest that these values themselves are due to errors of observation. It therefore seemed advisable to repeat the least-square solutions, rejecting the terms in $\delta\mu$. The results were.

From measures in x.

	Star A.	Star B.
$\delta x =$	$+0''\cdot025\pm0''\cdot023$	$+0''\cdot024\pm0''\cdot026$
$\pi =$	$+0\cdot094\pm0\cdot029$	$+0\cdot072\pm0\cdot032$
Probable error of unit weight	$\pm0\cdot056$	$\pm0\cdot063$

From measures in y.

$\delta y =$	$-0''\cdot018\pm0''\cdot023$	$-0''\cdot028\pm0''\cdot047$
$\pi =$	$+0\cdot106\pm0\cdot070$	$+0\cdot117\pm0\cdot139$
Probable error of unit weight	$\pm0\cdot061$	$\pm0\cdot122$

The representation of the observations is about as good as before, so that the idea that the large values of $\delta\mu$ are due to accidental error is confirmed. Combining these new values of the parallax with regard to their probable errors we have

Parallax of A	$+0''\cdot096\pm0''\cdot027$
B	$+0\cdot074\pm0\cdot031$

and for the mean of the two, with equal weights,

$$\pi = +0''\cdot085\pm0''\cdot021$$

This result differs from the one previously found by less than the probable error of either one. In the absence of certainty which of the two solutions is to be preferred we may perhaps best take the mean of the two, which gives

Parallax of γ Virginis $= +0''\cdot074\pm0''\cdot022$

as the best value, relative to the mean of the six comparison stars, which can be derived from our plates.

§ 14. The approximate discussion of the residuals for the comparison stars gives values for their parallaxes and proper

motions whose means (without regard to sign) are 0″·037 and 0″·051 respectively. These values appear to be due to errors of observation. If we assume that the comparison stars have no sensible parallax or proper motion, the probable error of a measured coordinate for one of them derived from one plate comes out ±0″·080. This is larger than the value previously found for the parallax star, so that it would appear that in this case the images taken through the gelatine patch of our colour-screen are better than those taken through the clear glass outside.

The probable error of a single image deduced from the comparison of the exposures on each plate with one another is ±0″·084, which would lead us to expect a probable error of ±0″·042 for a plate with four exposures. This is much less than the value given by comparison of different plates, so that it seems that in this series there is some sort of "plate error" which is nearly the same for all the images of one star on a plate.

Calculation of the effect of atmosphere dispersion on our results gives the following (when the seasonal variations of $d\beta$ are disregarded) :

Results from x.	Results from y.
$d\pi = -0{\cdot}005 d\beta$	$d\pi = +0{\cdot}004 d\beta$

so that we need fear no error from this source.

The average magnitude of our comparison stars is 8·9, corresponding to which Kapteyn gives the mean parallax 0″·006.

§ 15. The only previous determination of the parallax of *γ Virginis* known to the writer is a spectroscopic one by Belopolsky. He finds (*A.N.* 3510) that the relative velocity of the two components is 0·278 geographical miles per second, with a probable error of about ±0·1 g.m. With Doberck's elements of 1881 this gives $\pi = 0''{\cdot}051$. Owing to the uncertainty of the inclination of the orbit of the binary (given by different computers as from 31° to 37°) and that of the observed radial velocities of the two stars the probable error of the above value must be considerable. The agreement with the results of the present investigation is as good as there is any reason to expect.

§ 16. We may conclude by deriving from our parallaxes such information as we can get concerning the brightness, mass, &c. of the stars. In dealing with the brightness of stars the writer would suggest that Professor Kapteyn's conception of the "absolute magnitude" of a star should be generally used. By the absolute magnitude of a star Professor Kapteyn denotes the magnitude which it would appear to have at such a distance that its parallax was 0″·1. If m is the star's observed magnitude and π its parallax, we have then for the absolute magnitude m_0

$$m_0 = m + 5 - 5 \log \pi$$

In calculating this and similar quantities the relative parallax already found for our stars should be corrected by adding the probable mean parallax of the comparison stars.

We thus obtain for Lalande 21185 $\pi = +0''.351$ which, with the magnitude 7·3 and proper motion 4·77, gives

Absolute magnitude 10·0

The Sun's absolute magnitude is given by Kapteyn as 5·5, so that the star is 4·5 magnitudes fainter than the Sun, and gives about $\frac{1}{60}$ as much light.

The velocity of the star at right angles to the line of sight is 65 kilometres per second, with a probable error (so far as the present determination of the parallax is concerned) of about 3 km.

For γ *Virginis* we find the absolute magnitude of the two stars taken together to be 2·4. The two components are equal in brightness, so that the absolute magnitude of each one of them is 3·2 ; that is, each of them gives about nine times as much light as the Sun. The velocity of the system at right angles to the line of sight is 34 km. per second, while from Belopolsky's observations the velocity in the line of sight is 21 km., and the star is approaching us. This would make the velocity of the system in space 40 km. per second in a direction inclined about 60° to the line of sight. These values are, however, somewhat uncertain.

Using See's elements for the binary system ($a=3''.99$, P=194 years, $e=0.90$) we find

Major axis of orbit	= 50	astronomical units
Distance of stars at periastron	5	„
„ at apastron	95	„
Mass of system	3·3	

Auwers and Lewis have found that the masses of the two components are nearly equal, and so each of them must be about 1·6 times as massive as the Sun, whereas they each give about nine times as much light.

These stars must therefore be either less dense than the Sun or have a greater surface brightness, which accords well with the fact that their spectra are of the first type.

§ 17. In conclusion I wish to express my hearty thanks to the Director and staff of the Cambridge Observatory for the use of its instruments and of all its privileges, and for their cordial interest in the work ; and in particular to Mr. A. R. Hinks for much valuable comment and criticism, and especially for taking a large number of plates for me while I was disabled by a long illness.

Cambridge Observatory : 1905 *June* 9.

The Great Cluster in Hercules. By W. E. Plummer, M.A.

Some time since I received from Professor G. Hale, the late Director of the Yerkes Observatory, a photograph of the cluster in *Hercules*, taken with the large refractor. This plate is labelled: "Messier 13, photographed with the 40-inch Yerkes refractor, August 15, 1900. Exposure four hours, $8^h\ 30^m$–$12^h\ 30^m$ Central Standard Time, with double-slide plate-holder, yellow screen, isochromatic plate. (Signed) G. W. Ritchey." Having recently measured this plate, it seemed desirable to compare the results with what earlier measures existed, and to trace, if possible, any change in the relative coordinates of the stars forming the group. Such an inquiry is no doubt premature, but it might serve to show whether ten or a hundred years were needed in order to conduct such investigations with success.

At the time of the arrival of the plate I had no suitable measuring machine, but Professor Turner kindly placed at my disposal one that had been used at Oxford in the measurement of the plates of the International Chart. The American plate is of the ordinary quarter-plate size and without any réseau lines; while the Oxford machine, without a special adaptor, is only available for plates of the size of those recommended for the International Chart, and the method of measurement requires the employment of a réseau. It was therefore necessary to transfer the American negative to a plate of convenient size and marked with the réseau. For this service I was again indebted to the Oxford Observatory. The copy appears to have been very successfully effected. It might have been anticipated that in the two transfers necessary to produce a fresh negative many images of the fainter stars would be lost; but this is not so. In the case of the faintest stars visible on the original negative, I could generally trace them on the copy by the aid of allineation, and in only a few cases was the image too faint for measurement.

This transferred negative has been measured in two reversed positions of the plate, and with the scales and method of measurement employed the greatest accuracy attainable is the thousandth part of the distance between two réseau lines. Owing to the great focal length of the Yerkes refractor, the angle subtended between two of these lines is approximately $53''$, so that practically the measures have been made to $\frac{1}{20}''$. The diameters of the images were also measured, to determine the magnitude of the stars; but, as will be seen in the sequel, this portion of the work is not so satisfactory as could have been wished. The total number of objects measured is 2131, distributed over an area of eleven minutes square. The law of distribution throughout this area will be considered later.

To reduce the measurements to standard coordinates a certain number of fiducial stars is necessary, and within the small area

of the American negative, meridian places of these do not exist. It was therefore necessary to connect the Yerkes plate with one embracing a larger area. This plate was taken by Mr. H. C. Plummer and is discussed in the *Monthly Notices* of 1904, November. The reductions of the measures have been based upon the positions of the stars there given. Of the seventy stars whose places are recorded some sixty-two are common to both plates; but in a few instances the stars selected at Oxford for measurement were found to have a companion so close that the two images would, from the shorter focal length, naturally coalesce, and the centre of the combined image would not correspond to the centre of either, or necessarily to the mean of the two. On this ground seven stars were excluded from the comparison; and this process of rejection, for the reasons mentioned, might have been carried further with advantage. An inquiry, conducted in the usual way, showed that the Liverpool (L) measures could be connected with the Oxford (O) by means of the formulæ

$$x_0 = +0\cdot89584x_L + 0\cdot00061y_L - 8\cdot0234$$
$$y_0 = -0\cdot00033x_L + 0\cdot89556y_L - 14\cdot9226$$

Besides these Oxford measures, differential coordinates of a number of stars have also been given by Professor Scheiner, in his paper entitled "Der grosse Sternhaufen im *Hercules* Messier 13 nach Aufnahmen am Potsdamer photographischen Refractor" (*Abhandlungen Königl. Preuss. Akad. der Wissenschaften zu Berlin*, 1892). The places given in this paper are referred to a star, approximately in the centre of the group, whose coordinates for 1891·0 are quoted as

$$\alpha = 16^h\ 37^m\ 46^s\cdot85 \qquad \delta = +36^h\ 40^m\ 22^s\cdot9$$

This position, reduced to 1900·0, differs from the assumed centre of the Oxford plate by (P—O)

$$x = +1'\ 13''\cdot58$$
$$y = -\ 40\cdot40$$

For the purposes of comparison these constants have been applied to the Potsdam measures, together with the small differential corrections due to precession, in order to refer the measures to the equinox 1900·0. This was certainly not the best method of comparing the different sets of measures; but as it was the course that was actually pursued the results are set down here, as they show both the character of the agreement that may be expected between the measures and the necessity of adopting a more legitimate and accurate method of comparison: —

TABLE I.

Comparison of Measures of Potsdam, Oxford, and Liverpool.

Potsdam No.	Liverpool x 1900.	L−P.	Oxford No.	L−O.	Liverpool y 1900.	L−P.	L−O.
	′ ″	″		″	′ ″	″	″
7	−3 55·27	−0·26	7	−0·07	+4 52·68	−0·37	0·00
9	−3 52·57	+0·25	8	−0·13	−5 13·56	−0·58	−0·54
10	−3 48·71	−0·09	9	−0·05	+ 55·05	+0·04	+0·63
14	−3 29·89	+0·14	11	−1·45	−2 41·83	−0·70	−0·19
27	−2 57·83	+1·54	13	+0·61	−1 53·97	−0·13	+0·45
29	−2 56·45	+0·17	14	+0·19	−3 52·07	−0·78	−0·17
30	−2 54·16	+0·24	15	+1·40	+2 35·55	−0·80	+0·09
34	−2 39·68	−0·02	16	−0·5	− 31·65	−0·04	+0·39
47	−2 7·84	+0·03	17	−0·82	+1 46·49	−0·56	+0·89
48	−2 4·29	+0·44	18	+0·09	−3 0·13	−0·45	−0·13
51	−1 57·43	+0·51	19	−0·01	−2 45·87	−0·46	−0·69
56	−1 38·20	+0·63	21	+0·92	−5 41·34	−0·80	−0·66
63	−1 26·50	+0·06	22	−0·34	− 31·82	−1·58	−0·62
68	−1 19·91	−0·19	23	+0·85	+5 25·84	−0·42	+0·52
72	−1 15·25	+0·28	24	+0·71	−3 3·81	−0·47	−0·21
92	− 51·78	+0·05	25	+0·18	+3 24·73	−0·12	+0·73
103	− 43·70	+0·08	26	+0·22	+1 11·58	−0·90	+0·12
109	− 39·85	+0·20	27	−0·01	−2 33·00	−0·43	−0·18
127	− 26·87	+0·10	28	−0·35	− 1·90	−0·53	+0·20
129	− 23·54	+0·19	29	+0·34	− 57·21	−0·80	−0·21
130	− 23·04	+0·39	30	−0·48	−3 49·30	−0·16	+0·32
148	− 8·16	+0·47	31	−0·54	−2 24·22	−0·30	+0·44
216	+ 19·76	+0·14	32	−0·16	− 43·74	−0·44	−0·12
227	+ 22·55	+0·32	33	−0·07	−3 21·97	−0·18	+0·35
234	+ 28·27	+0·13	34	−0·05	+2 19·72	+0·03	+0·76
287	+ 49·05	+0·08	35	+0·27	− 2·71	−0·63	−0·49
288	+ 50·10	+0·55	36	+0·06	−5 52·50	−0·05	−0·24
296	+ 52·11	+0·29	37	−0·09	−2 54·67	−0·31	−0·07
418	+1 23·44	+0·74	38	−0·08	+ 16·72	−0·68	+0·88
479	+1 40·43	+0·70	40	+0·59	−2 4·87	−0·12	+0·05
507	+1 49·42	+0·81	41	+0·76	−2 6·97	−0·96	−0·73
529	+1 53·87	+0·35	42	−0·37	−5 59·43	+0·02	+0·39
594	+2 15·02	+0·01	43	−0·94	+2 33·56	−0·07	+0·02
603	+2 16·48	+0·03	44	−0·20	+1 37·46	−0·51	−0·28
602	+2 16·58	+0·15	45	−0·46	+1 24·89	−0·52	−0·07
642	+2 32·71	+0·03	46	−0·29	−2 36·24	−0·09	+0·18

Potsdam No.	Liverpool x 1900.	L−P.	Oxford No.	L−O.	Liverpool y 1900.	L−P.	L−O.
	′ ″	″		″	′ ″	″	″
654	+2 39·46	+0·23	47	+0·22	− 5·59	−0·55	−0·19
726	+3 11·37	+0·45	48	+0·21	−4 18·48	+0·08	−0·12
727	+3 11·88	+0·44	49	+0·12	−4 52·52	−0·15	+0·34
731	+3 14·03	+0·34	50	+0·35	+1 8·72	−0·70	+0·32
739	+3 21·11	+0·24	51	−0·01	+4 43·74	−0·90	−0·60
749	+3 34·33	+0·20	53	+0·01	−2 5·58	−0·19	+0·18
748	+3 34·15	+0·81	54	−0·41	−5 54·40	−0·45	+0·26
771	+4 5·18	+0·50	55	−0·34	−1 12·02	−0·54	+0·34
774	+4 14·24	+1·37	56	+0·56	+ 26·78	+1·37	+0·50
778	+4 18·76	+0·06	57	−0·14	+4 51·56	−0·51	−0·28
781	+4 30·94	+0·52	58	+0·70	+1 27·40	−0·56	−0·26
785	+4 36·99	+0·33	59	−0·21	−3 13·61	−0·63	+0·07
786	+4 37·78	−0·04	60	−0·38	+5 19·78	−0·29	−0·14
795	+5 2·93	+0·20	61	+0·23	− 49·26	−0·44	−0·12
799	+5 18·38	+0·58	62	−0·10	−5 1·65	−0·11	−0·45
805	+5 37·44	+0·25	63	0·00	+3 13·11	−0·63	−0·57
815	+6 44·10	+0·14	64	+0·78	+1 0·21	−0·49	+0·09
821	+7 6·50	+1·05	65	−0·10	−6 10·61	−0·37	−0·29

Note.—Star No. 30 Potsdam has another star with which it might coalesce. No. 60 is ill-formed on the Yerkes plate, and has not been used in deriving the plate constants. No. 418 is described by Scheiner as "neblig." It has a small star quite close. No. 748 has the same note, and is similarly circumstanced. No. 774 is marked "Duplex oder nebelknoten." This is an instructive case. There is another star distant 1″·36 in x and 2″·69 in y. Neither star agrees very well with the Potsdam place, but the mean of the two would be satisfactory.

The few changes of sign in the residuals (L−P) disclose a systematic difference between the two sets of measures, while the uniformity in the amount offers little hope of tracing any effects of motion among the stars. The larger part of the discrepancies may be removed by a slight alteration in the assumed mean distance between the origin of coordinates on the Liverpool (or Oxford) and Potsdam plates. The mean difference between Liverpool and Potsdam, as determined from these fifty-four stars, is (L−P)

x	...	...	+0″·32
y	...	...	−0·41

It is not difficult to suggest various ways in which this difference might arise; but that it is a real difference and does not arise from the use of the outlying and brighter stars of the group is shown by a more extended comparison between the stars that have been measured at Potsdam and Liverpool. I proposed

to compare the whole of the Potsdam measures with those derived from the Yerkes plate, but in the denser part of the cluster it has not always been possible to identify with certainty the star that has been measured by Professor Scheiner; and, in addition to these instances of defective identification, some other stars have been rejected because they are accompanied by a bad note implying uncertainty in the original measures. The total number of stars between which it seemed safe to institute comparisons is 560, and the general result is shown in Table II., where a zone is to be understood as an arc of declination between two consecutive réseau lines on the copy of the Yerkes plate.

TABLE II.

(L−P).

Zone.	Approx. Distance of Centre of Zone from Centre of Group.	Number of Stars Compared.	Mean Difference of x.	Mean Difference of y.
	′ ″		″	″
1	−5 44	12	+0·44	−0·20
2	−4 50	14	+0·57	−0·17
3	−3 57	28	+0·43	−0·22
4	−3 3	62	+0·32	−0·33
5	−2 9	86	+0·29	−0·37
6	−1 15	103	+0·32	−0·39
7	−0 22	95	+0·16	−0·52
8	+0 31	76	+0·19	−0·62
9	+1 25	37	+0·09	−0·64
10	+2 19	23	+0·14	−0·63
11	+3 13	12	+0·16	−0·34
12	+4 6	6	+0·11	−0·54
13	+4 59	4	+0·30	−0·35

Here again the mean difference is +0″·27 and −0″·41 in x and y respectively, practically the same as before, with some indications of regular systematic progress. But the systems by which the measures have been reduced in the two cases are essentially different; and before deciding to what extent the discrepancy in the measures is real it will be necessary to reduce the measures to some uniform plan. There are two methods which seem available: either to introduce a scale value into Dr. Scheiner's work which shall be the same as that employed in the Oxford measures, and to re-reduce the position of his normal star, or to derive the constants of the Potsdam plate *de novo*, and in the same manner and from the same stars that were used in deriving the constants of the Yerkes plate.

With regard to the first proposal it is to be remarked that Dr. Scheiner had at his command the places of seven stars out of the Lund zones, and these places, or some of them, have been used for the determination of the scale. From stars A.G.C. Lund 6829 and 6863 he has derived one value of the scale or réseau interval, and from Lund 6836 and 6868 he has derived

another; and since he has two plates taken on consecutive evenings he obtains four results :—

	From the First Plate.	From the Second Plate.
1st pair of stars	299″·87	299″·86
2nd pair of stars	299·96	299·97
Mean	299·92	299·92

This mean result looks very satisfactory; but when we introduce the corrections (*Monthly Notices*, 1904 November, vol. lxv. p. 81) to the Lund stars, we get quite as accordant results though the means differ noticeably :—

	From the First Plate.	From the Second Plate.
1st pair of stars	299″·843	299″·821
2nd pair of stars	299·868	299·888
Mean	299·855	299·855

The position of the star of reference has been derived from A.G.C. Lund 6829 and 6868, and from A.G.C. Lund 6836 and 6857. The measured angles Δ, given on page 19, I have reduced in the ratio of 299·855 : 299·92; but I have used unchanged the angle p (the angle between the lines drawn from the star of reference to one pair of the Lund stars), and have again brought in the corrections to the places of the Lund stars given in the *Monthly Notices* already referred to. The positions of the star of reference as given by Dr. Scheiner are

	Plate I.	Plate II.
1st pair	$\alpha = 249^\circ\ 27'\ 39''{\cdot}3$ $\delta = 36^\circ\ 41'\ 5''{\cdot}0$	$\alpha = 249^\circ\ 27'\ 35''{\cdot}1$ $\delta = 36^\circ\ 41'\ 1''{\cdot}2$
2nd pair	$\alpha = 249\ 27\ 39{\cdot}3$ $\delta = 36\ 41\ 4{\cdot}7$	$\alpha = 249\ 27\ 34{\cdot}3$ $\delta = 36\ 41\ 0{\cdot}8$

The new values for the same star are

1st pair	$\alpha = 249^\circ\ 27'\ 39''{\cdot}21$ $\delta = 36^\circ\ 41'\ 4''{\cdot}71$	$\alpha = 249^\circ\ 27'\ 34''{\cdot}99$ $\delta = 36^\circ\ 41'\ 0''{\cdot}52$
2nd pair	$\alpha = 249\ 27\ 39{\cdot}69$ $\delta = 36\ 41\ 4{\cdot}90$	$\alpha = 249\ 27\ 34{\cdot}75$ $\delta = 36\ 41\ 0{\cdot}69$

The mean difference in α is $+0''{\cdot}27$ (or in $x+0''{\cdot}22$) and in $\delta-0''{\cdot}41$; a very close agreement with the quantity shown to be necessary to remove the constant error between the two sets of measures.

The same result is practically arrived at if the Potsdam measures are compared, not directly with the Lund stars, but with the places derived in the *Monthly Notices*. Taking the same list of stars as that previously employed, page 803, the difference between Oxford and Potsdam is given by the expression

$$x_0 - x_P = +0''{\cdot}029x_P - 0''{\cdot}107y_P + 1'\ 13''{\cdot}87$$
$$y_0 - y_P = +0''{\cdot}092x_P - 0''{\cdot}042y_P - \ \ 40''{\cdot}83$$

These formulæ include the effects of (1) difference of adopted plate centres, (2) difference of central meridian, (3) difference of mean equinox, (4) changes in (*a*) scale value and (*b*) orientation

arising from the different systems of reduction. It is unnecessary to seek in detail to what extent the total coefficients are accounted for by these sources of difference, so far as they can be separately estimated. But the coefficients $+0''\cdot029$ and $-0''\cdot042$, which represent the total difference of scale value (per minute of arc) in the direction of the two coordinates, call for remark. No geometrical reason can be suggested for the difference, and the magnitude of the discrepancy ($0''\cdot071$) is too great to be attributed with any assurance to the influence of accidental error. Hence, although the proof is by no means convincing, there is a slight suggestion that the form of the cluster as a whole is changing either by an expansion in the direction of x or a contraction in the direction of y.

Introducing these corrections into the Potsdam Catalogue, the residuals in Table II. have the following values :

Zone.	Mean Diff. of x.	Mean Diff. of y.	Zone.	Mean Diff. of x.	Mean Diff of y.
	$''$	$''$		$''$	$''$
1	$-0\cdot06$	$+0\cdot02$	8	$-0\cdot05$	$-0\cdot14$
2	$-0\cdot14$	$+0\cdot07$	9	$-0\cdot06$	$-0\cdot07$
3	$-0\cdot07$	$+0\cdot04$	10	$+0\cdot03$	$-0\cdot06$
4	$+0\cdot02$	$-0\cdot11$	11	$0\cdot00$	$+0\cdot25$
5	$-0\cdot04$	$+0\cdot19$	12	$+0\cdot01$	$+0\cdot04$
6	$+0\cdot01$	$-0\cdot02$	13	$+0\cdot06$	$+0\cdot27$
7	$+0\cdot03$	$+0\cdot05$	...	...	...

The smallness of the residuals shows, as was expected, that the interval of time is far too short to detect any relative motion in the stars composing the system. But the method of grouping the stars is not without its objections. It is probable that in a spherical cluster, seen projected on a plane, many of the stars apparently near each other are on opposite sides of the system and moving in opposite directions ; consequently in such cases the motions would have a tendency to counteract each other.

Magnitude.

The determination of magnitude is one of the least satisfactory parts of the inquiry. Not only is there wanting sufficient data to assist in the conversion of the measured diameters into conventional magnitudes, but the measurement of the diameters themselves is not very satisfactory. So long as the images present a well-blackened area the measurement is fairly easy, but in the faintest stars the blackening is not continuous. They consist of a certain number of darkened points which do not coalesce, and there is probably a tendency to make the measurements too large. Or if the exposure had been continued, producing still fainter stars on the plate, the images that have been measured would not have increased in diameter, but the internal parts would have become blacker. Practically, therefore, it is to

be feared that there are two scales, one referring to the brighter, the other to the fainter stars.

Inasmuch as the total number of stars measured is roughly $2\frac{1}{2}$ times that of Scheiner's Catalogue, it might be assumed that the Yerkes plate registers the positions of stars one magnitude fainter than those on the Potsdam plates. This would imply the fifteenth magnitude; but it is hardly probable that a law which has been found to hold roughly among the brighter and scattered stars in the heavens would be maintained here in orders of magnitude which have not hitherto been considered. The increase of number with decrease of magnitude must break down some time, and apart from this, there is some evidence that the distribution of stars of various magnitudes is not uniform. In Scheiner's catalogue the stars which are described as faintest are not those which occur most frequently. He has adopted practically seven classes of magnitudes, and in the case of those stars which it is possible to compare with the Liverpool measures the percentages in each class are as follows :

Magnitude.	Percentage.	Magnitude.	Percentage.
12·4	5	13·0	26
12·5	7	13·5	20
12·7	10	14	17
12·8	15		

This peculiarity in the falling off in numbers is not shown on the Yerkes plate, but another is manifest for which there appears to be no explanation in the way of observation, and which points with equal force to a want of uniformity in the distribution of the stars in each magnitude. Arranging the stars in order of diameter in successive groups, varying by five units (which corresponds approximately to $0''{\cdot}3$) we get the following numbers in each group :

TABLE IV.

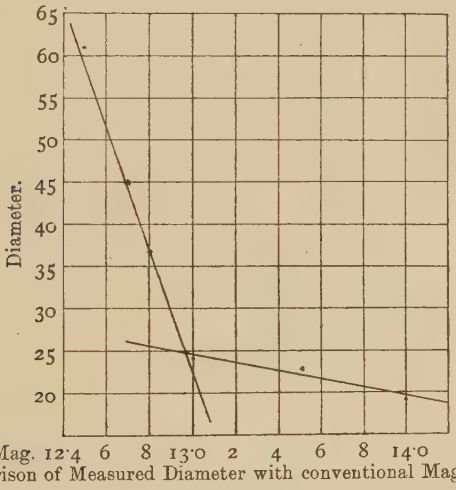
Number of Stars in each Zone, arranged according to Diameter.

Distance from Centre.	Under 15.	15–20.	20–25.	25–30.	30–35.	35–40.	40–45.	45–50.	50–55.	55–60.	Over 60.	Objects not measurable.	Total.
−5·44	7	5	2	1	...	...	1	...	...	...	1	...	17
−4·50	18	4	...	3	1	2	1	3	...	1	...	...	33
−3·57	12	9	3	3	3	1	1	...	...	...	2	4	38
−3·3	24	11	6	5	2	3	2	1	...	1	5	11	71
−2·9	45	32	9	4	7	7	4	5	3	1	6	15	138
−1·15	144	64	33	10	11	13	12	8	3	5	13	22	338
−0·22	217	108	67	41	21	42	22	21	6	21	29	38	633
+0·31	214	92	45	30	10	26	13	7	6	3	16	10	472
+1·25	92	39	22	7	3	9	13	3	1	3	2	1	195
+2·19	45	15	10	3	...	4	4	1	2	2	4	...	90
+3·13	17	19	4	1	...	3	1	...	1	...	1	...	47
+4·6	11	11	1	5	1	1	...	1	...	...	1	...	32
+4·59	10	1	...	3	1	...	...	...	...	...	...	...	15
+5·52	5	2	2	1	...	...	...	1	1	...	...	...	12
Total	861	411	204	117	60	111	75	51	23	37	80	101	2131

While the rate of increase is fairly well marked in the lower numbers there are noticeable discrepancies among the higher magnitudes, and to some extent the table demonstrates the same anomaly as that found in Scheiner's catalogue. I have attempted to determine the average diameter that corresponds to Scheiner's assigned magnitudes, as it would be obviously desirable to continue Scheiner's scale of magnitudes. The results are as follows :

Scheiner's Magnitude.	Yerkes Diameter.	No. of Stars.
12·4	65·6	28
12·5	61·4	38
12·7	44·9	56
12·8	37·1	78
13·0	24·4	139
13·5	22·9	106
14	19·1	90

Scheiner has marked two stars as of 12·2 and 11·7 magnitudes respectively ; but the diameters of these fall below the average of 12·4, and two others are described as 13·8 magnitude. These



Comparison of Measured Diameter with conventional Magnitudes.

also have been excluded from the list. If these magnitudes and diameters be plotted to scale the result points, as I have suggested, to the existence of two scales, or two distinct methods of measuring the diameter. It does not seem practical to draw a curve, even if a curve were legitimate, through the points. The diameters of the brighter stars lie on one straight line, and the diameters of the fainter on another. This is shown on the accompanying diagram where a linear relation apparently exists between both the brighter and the fainter stars. Four hours'

exposure with the Yerkes telescope would undoubtedly impress the images of fainter stars than those of the fourteenth magnitude, but the effect of the yellow screen is not known. But considering that Scheiner must have included some stars below the fourteenth magnitude in his faintest group, and after allowing for the over-estimation of the diameters of the faintest stars on the Yerkes plate, it seems not improbable that the number of stars brighter than the fifteenth magnitude does not greatly exceed 2,000.

The Distribution of Stars in the Cluster.

A question of greater interest seems to be to attempt to determine the law of density in the cluster itself, and the larger number of stars with which we have to deal than were at the command of Professor Scheiner makes the attempt a little more promising. There is, of course, some difficulty in deriving the number of stars in a definite area of various distances from the centre, but a tolerable approximation has been effected by dividing each square of the réseau (53″ of side) into four equal portions, and counting the number of stars in each quarter. These numbers were plotted in a diagram from which could be obtained the average number in a small square at uniform distances from the centre. At the centre of the cluster the number of stars in a square whose side was approximately twenty-seven seconds was sixty-five, and the cluster has been assumed to extend in the directions x and y to six minutes from the centre. At this distance about one star was found in each réseau square, and therefore the influence of the cluster may be supposed to have disappeared. In this way we have the apparent distribution of the stars when projected on a plane at right angles to the line of sight.

On the assumption of general radial symmetry let $f(r)dv$ be the number of stars in a small volume dv situated at a distance r from the centre of the cluster. Let x be the projection of r on the plane at right angles to the line of sight, and θ the angle between r and x. Then if a cylinder of small section da be taken through the cluster with its axis parallel to the line of sight and at a distance x from the centre of the cluster, the number of stars to be seen within it is

$$\int f(r)da d(x \tan \theta)$$

where $x = r \cos \theta$ and the limits of the integral correspond to the boundary of the cluster. Eliminating θ and equating to $\phi(x)da$, the number of stars within the area da in the apparent distribution, we get

$$\phi(x) = 2\int_x^1 rf(r)\frac{dr}{\sqrt{r^2-x^2}}$$

the radius of the cluster being taken to be unity. If r be eliminated instead of θ, we get

$$\phi(x) = 2x\int_0^{\theta'} \sec^2\theta f(x\sec\theta)d\theta$$

Here the upper limit θ' corresponds to the boundary and is a function of x when the radius of the cluster is finite. If the radius can be considered infinite the upper limit becomes $\frac{1}{2}\pi$. Hence if $f(r)$ can be expressed in the form $\Sigma a_n r^{-n}$, $\phi(x)$ has the corresponding form $\Sigma A_n x^{-n+1}$, where

$$A_n = 2a_n\int_0^{\frac{\pi}{2}} \cos^{n-2}\theta d\theta$$

When the law of density is expressed by one term only, this can easily be found ; for in this case

$$\log\phi(x) = \log A_n - (n-1)\log x$$

which gives a straight line when plotted with log x and log $\phi(x)$ as coordinates. In the present case the graph shows distinct curvature, indicating that the law of density cannot be well represented by this simple form. The effect of a finite boundary to the cluster was next considered, but without an entirely satisfactory result. It seems probable that if the law of density is of this type, it requires more than one term for its expression.

When the radius of the cluster is supposed to be finite, a simple type of the law of density is represented by some positive power of the distance from the boundary. The radius being taken as unity, this law is expressed by $f(r) = (1-r)^n$.

In this case

$$\phi(x) = 2\int_x^1 r(1-r)^n \frac{dr}{\sqrt{r^2-x^2}}$$

$$= 2\left[(1-r)^n\sqrt{r^2-x^2}\right]_x^1 + 2\int_x^1 n(1-r)^{n-1}\sqrt{r^2-x^2}dr$$

where the first term on the right vanishes at both limits. Let

$$Z_n = \int_n^1 (1-r)^n \frac{dr}{\sqrt{r^2-x^2}}$$

The equality of the above integrals shows that

$$Z_n - Z_{n+1} = n[Z_{n+1} - 2Z_n + (1-x^2)Z_{n-1}]$$

or

$$(n+1)Z_{n+1} - (2n+1)Z_n + n(1-x^2)Z_{n-1} = 0$$

Let

$$Y_n = Z_n - Z_{n+1}$$

so that

$$-(n+1)Y_n + nY_{n-1} - nx^2Z_{n-1} = 0$$

$$-nY_{n-1} + (n-1)Y_{n-2} - (n-1)x^2Z_{n-2} = 0$$

Then the result of eliminating the Z functions becomes

$$(n^2-1)Y_n-n(2n-1)Y_{n-1}+n(n-1)(1-x^2)Y_{n-2}=0$$

Now $\phi(x)=2Y_n$ and

$$Y_1=\tfrac{1}{2}a-\tfrac{1}{2}b,\ Y_2=\tfrac{1}{3}(2x^2+1)a-b$$

where

$$a=(1-x^2)^{\frac{1}{2}},\quad b=\tfrac{1}{2}x^2\log_e(1+a)/(1-a)$$

When $x=0$, $Y_1=\frac{1}{2}$ and $Y_2=\frac{1}{3}$, which shows that $Y_n=\dfrac{1}{n+1}$, for the solution of the difference equation in this case is clearly

$$Y_n=A/(n+1)+Bn$$

Hence if we put $\phi_n(x)=(n+1)Y_n$ we make the central density in the apparent distribution unity. The difference equation becomes

$$(n-1)\phi_n-(2n-1)\phi_{n-1}+n(1-x^2)\phi_{n-2}=0$$

with the initial values

$$\phi_0=a,\ \phi_1=a-b,\ \phi_2=(2x^2+1)a-3b$$

These can be calculated numerically for convenient intervals of x, and the values of ϕ_3, ϕ_4 . . . can be easily deduced in succession by means of the difference equation. When the process has to be carried to considerable values of n, it is of course necessary to begin with a greater number of decimal places than are ultimately required on account of the accumulation of numerical error. In this way the following table has been calculated :

n/x.	0·0.	·1.	·2.	·3.	·4.	·5.	·6.	·7.	·8.	·9.	1·0.
0	1·000	0·995	·980	·954	·917	·866	·800	·714	·600	·436	·000
1	1·000	·965	·888	·785	·666	·537	·405	·275	·156	·057	·000
2	1·000	·925	·783	·620	·458	·311	·190	·097	·037	·007	·000
3	1·000	·880	·679	·477	·305	·174	·085	·033	·008	·001	·000
4	1·000	·831	·582	·362	·200	·095	·038	·011	·002		
5	1·000	·782	·494	·271	·129	·051	·016	·003			
6	1·000	·733	·417	·202	·083	·027	·007				
7	1·000	·684	·350	·149	·052	·014	·003				
8	1·000	·637	·292	·109	·033	·007	·001				
9	1·000	·592	·243	·080	·021	·002	·001				
10	1·000	·549	·202	·058	·013						
11	1·000	·509	·167	·042	·007						
12	1·000	·470	·137	·030	·004						·000

When these values are plotted they give curves which represent the general character of the observed distribution of density. For a good agreement a large value of n seems necessary. As n increases the curves tend to similarity, and the uncertainty in the radius of the cluster makes it impossible to attach any great weight to the determination of n. With the diameter already mentioned a fair representation is obtained by assuming $n = 11$. Unfortunately it is not easy to show the comparison graphically except on a very large scale, as the variation from straight line is small in the greater part of the curve, and the various curves have a tendency to coalesce. But assuming a central density of sixty-five stars in a unit of space, the number of stars read from the curves can be compared numerically with the values computed from the above table.

Distance from Centre.	No. of Stars Observed.	No. of Stars $n=8$.	No. of Stars $n=9$.	No. of Stars $n=10$.	No. of Stars $n=11$.
″ 0	65·0	65·0	65·0	65·0	65·0
45	33·4	41·4	38·5	35·7	33·1
93	12·7	19·0	15·8	13·1	10·9
136	5·0	7·1	5·2	3·8	2·7
188	2·3	2·2	1·4	0·8	0·5
258	1·1	0·5	0·1		

The agreement is not very satisfactory in any case, but it seems safe to say that while n may be as high as 11, it is not less than 8. Professor Geo. E. Hale has offered me a plate on which the number of images recorded is much larger than on the plate here discussed, and I hope to return to this question again.

The discussion as far as it goes affords an example of the spirit of devolution of which we have heard something of late, and I acknowledge my indebtedness to the Directors of the Yerkes and Oxford Observatories, and to others, who have enabled me to carry out this investigation.

1905 *June* 5.

The Solar Rotation Period from Greenwich Sun-spot Measures, 1879–1901. By E. Walter Maunder and A. S. D. Maunder.

1. *Material Employed.*

The material employed in the following paper is much more ample than has been used in any previous discussion of the solar rotation. It is drawn from the Photoheliographic Results published in the annual volumes of the *Greenwich Observations*, and especially from the ledgers of sun-spots. From the 4700 spot-groups catalogued in these results for the twenty-three years (1879–1901) we have taken every group which persisted

for six or more days—1872 in number. A very few groups of this duration were excluded, because the record was in some way defective. For the sake of symmetry the last groups of the cycle, which expired in 1879, and the first groups of the cycle, which began in 1901, have been left out of the discussion, which is thus limited to the two complete cycles. These two cycles will be referred to hereafter as cycle 1 and cycle 2—cycle 1 attaining its maximum in 1883, and cycle 2 in 1893. It may be noted here that when sun-spots are treated, as in this discussion, in narrow zones of latitude, there is (broadly speaking) not the slightest possibility of ambiguity as to which of the two cycles a given group belongs. The break between the two cycles in any particular latitude generally lasts for something like three full years during which no spots whatsoever, not even the most minute, appear in that particular zone. The time when this break takes place differs for different zones; but for any particular zone this entire cessation of activity is one of the most unmistakable characteristics of solar variation.

The material has been used in two ways. First, each apparition of a spot-group has been used independently of any other apparition of the same. Second, the spot-groups have been carefully examined for cases of return, and where it appeared clear that the same group has returned a second time or more frequently, without any temporary disappearance or subsidence, such a long-continued group has been treated as an entity throughout. But in both methods the entire spot group has been taken as a whole on each day of observation, the centre of gravity of the entire group being taken as the position of the group on that particular day. There has been no selection of spots because they seemed to be steady in motion or regular in shape, no rejection because of unsteadiness or irregularity. The only criterion for the inclusion of a group in the discussion has been that it lasted for six consecutive days; subject to the caution that when a group on a certain day was close to the limb, and there was reason to fear from a marked and sudden change of area or position that the entire group was not in view, that day was not used. If on the first or last day of observation the group was within 70° of the central meridian, then the group was retained, no matter whether the measures appeared accordant or not. If the distance from the central meridian was much greater than 70° on any day, then that day's measures were only retained if there was clear indication that the whole of the group was still within the visible hemisphere.

The method of reduction employed was very simple, but seemed sufficiently effective for the purpose in view. The mean of the positions of the group on the first three days was taken as the first position of the group; the mean of the positions on the last three days as the second position; the difference between the two longitudes thus given divided by the number of days and parts of a day gave the mean daily drift, eastward or west-

ward according to its sign. Thus a group observed on six consecutive days would have its mean position determined for the second and fifth days, giving an interval of three days. The longest period during which a group can be observed during a single apparition is fourteen days, and it was of course extremely rare under these circumstances that both the first and the last observations could be regarded as complete. A fourteen-day group, where all the fourteen days could be used, would thus give an interval of eleven days.

In the case of the long-continued spots, that is to say, of spots that have been seen during more than one apparition, the mean of the first five days of the first apparition has been compared with the mean of the last five days of the last apparition, wherever this was possible. As in the former case, days of observation when the group was imperfectly seen at either limb have not been included.

The term "solar rotation period" is not used in this paper in a rigid sense. Strictly speaking, the Sun can have but one rotation period, but an inquiry like the present, confined entirely to the apparent movements of spot-groups, necessarily deals with the resultant effect of such solar rotation period and of the motions of the groups, whether such motions are systematic, or irregular, or both. To speak, for example, of the solar rotation period as derived from a single group, possibly short-lived, and moving at an unusual speed, would be quite inexcusable if it were not clearly pointed out that the expression was used partly for brevity and convenience, and partly because the solar rotation period is, after all, much the most considerable factor, as well as the most permanent in the apparent movements of the spots upon the solar disc.

2. *Each Apparition treated separately.*

In the reduction of the measures of the solar photographs at Greenwich Carrington's value for the mean sidereal rotation period of the Sun has been adopted throughout. This period is 25·38 days, corresponding to a daily sidereal angular motion of 851′·06. The first operation in the discussion of the observations was to ascertain for each group the amount of its apparent daily drift in longitude as computed with this constant. The "drifts" thus obtained have been combined in various ways, the first attempt being to find an expression for the variation of the rotation rate in different solar latitudes. Table I. exhibits the result of treating the spots in zones of latitude each 5° wide. The centres of these zones have been taken $2\frac{1}{2}°$ apart, starting from the equator and proceeding in either direction. Thus every group has been used twice, except the groups in the two exterior half-zones. This is the only smoothing that has been used. The weighting has been strictly in proportion to the number of days' interval given by each group; that is to say, a

group lasting six days would have its mean positions taken for the second and fifth days; the interval would be three days and the weight would be 3.

TABLE I.

Rotation Period for Different Zones of Latitude each Five Degrees wide.

Latitude.	Number of Groups.	Weight. Groups × Days.	Apparent Mean Daily Drift in Longitude.	Mean Synodic Rotation Period.	Daily Sidereal Motion.	Mean Sidereal Rotation Period	Daily Sidereal Motion computed.	O−C.
			′	d	′	d	′	′
+35°	2	12	−46·3	28·97	804·8	26·84	821·7	−16·9
32½	3	21	23·6	28·11	827·4	26·11	828·4	− 1·0
30	11	71	18·7	27·94	832·3	25·95	834·7	− 2·4
27½	36	212	11·6	27·68	839·4	25·73	840·2	− 0·8
25	66	400	7·4	27·53	843·6	25·61	846·4	− 2·8
22½	115	703	− 0·4	27·29	850·7	25·39	851·7	− 1·0
20	167	1018	+ 7·6	27·02	858·6	25·16	856·5	+ 2·1
17½	188	1174	13·6	26·82	864·6	24·98	860·9	+ 3·7
15	241	1587	14·3	26·79	865·4	24·96	864·7	+ 0·7
12½	291	1934	15·8	26·74	866·9	24·92	868·0	− 1·1
10	246	1592	19·7	26·61	870·8	24·81	870·7	+ 0·1
7½	154	987	23·0	26·50	874·1	24·71	872·9	+ 1·2
5	99	629	26·0	26·41	877·0	24·63	874·4	+ 2·6
+ 2½	63	392	28·1	26·34	879·1	24·57	875·4	+ 3·7
0	41	256	25·2	26·43	876·2	24·65	875·7	+ 0·5
− 2½	70	453	23·3	26·49	874·4	24·70	875·4	− 1·0
5	148	957	20·6	26·58	871·7	24·78	874·4	− 2·7
7½	240	1570	18·2	26·66	869·3	24·85	872·9	− 3·6
10	289	1874	17·4	26·69	868·4	24·87	870·7	− 2·3
12½	294	1867	15·8	26·74	866·8	24·92	868·0	− 1·2
15	277	1740	12·8	26·84	863·8	25·00	864·7	− 0·9
17½	252	1573	8·6	26·98	859·6	25·13	860·9	− 1·3
20	188	1195	5·4	27·09	856·4	25·22	856·5	− 0·1
22½	110	696	+ 1·4	27·23	852·5	25·34	851·7	+ 0·8
25	69	422	− 5·6	27·47	845·5	23·55	846·4	− 0·9
27½	44	273	7·7	27·56	843·3	25·63	840·2	+ 3·1
30	24	150	11·8	27·69	839·2	25·74	834·7	+ 4·5
32½	9	50	26·8	28·23	824·3	26·21	828·4	− 4·1
−35	3	12	−33·7	28·49	817·4	26·43	821·7	− 4·3
All Spots*	1871	11915	+13·1	26·83	864·17	25·00	...	...
Carrington	...	...	...	27·275	851·06	25·38	...	...

* Each spot is, of course, only taken once in this summary.

TABLE II.

bers of Spot-groups, arranged in Zones of Latitude 5° wide, giving Different Rotation Periods.

c m .	Above +35°.	+35° to +30°.	+30° to +25°.	+25° to +20°.	+20° to +15°.	+15° to +10°.	+10° to +5°.	+5° to 0°.	0° to −5°.	−5° to −10°.	−10° to −15°.	−15° to −20°.	−20° to −25°.	−25° to −30°.	−30° to −35°.	Below −35°.
	...	...	...	...	...	...	...	...	...	...	1	..	...	...	...	...
	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
	...	...	...	...	...	...	...	...	...	...	2	2	...	...	...	...
	...	...	...	...	2	...	1	...	...	2	...	1	...	...	...	...
	...	...	...	...	...	1	1	...	2	1	2	1	...	...	...	...
	...	...	...	...	...	3	...	1	2	3	1	...	1	1	...	...
	...	...	...	...	2	2	3	1	1	1	3	2	...	...	...	...
	...	...	1	3	2	2	2	2	3	4	3	6	...	...	...	...
	...	...	...	1	3	5	4	4	2	6	6	1	1	...	...	...
	...	...	...	1	5	5	2	2	1	4	13	3	...	1	...	...
	...	...	1	3	4	12	7	1	3	5	5	8	2	1	...	...
	...	...	1	1	9	12	15	4	2	5	13	9	2	...	...	...
	...	...	...	4	6	12	10	10	2	12	14	9	8	1	...	...
	...	...	2	4	7	23	16	1	5	25	16	16	7	1	...	...
	...	...	1	5	15	21	14	10	6	19	24	14	4	...	...	...
	...	1	3	8	15	23	7	9	15	26	26	14	9	2	...	...
	...	...	2	3	14	31	26	9	7	20	30	16	7	4	1	...
	...	...	...	7	25	41	17	8	6	42	41	23	8	1	...	...
	...	...	1	13	25	39	15	...	5	26	37	38	12	3	...	...
	...	...	2	15	18	29	8	1	1	19	23	33	5	4	1	...
	...	...	3	12	22	7	...	...	2	5	13	19	14	4	...	1
	...	...	4	11	5	9	3	...	1	5	4	16	13	6	1	...
	...	...	4	5	4	4	1	...	1	5	4	2	3	7	...	...
	...	...	2	5	3	2	3	...	2	2	3	4	7	2	1	...
	...	...	3	5	2	1	...	...	1	2	4	3	5	3	1	...
	...	2	2	1	...	2	...	..	...	1	2	1	...	...	1	...
	...	...	1	2	...	...	...	...	...	...	2	6	1	1	1	...
	...	...	1	3	...	1	...	...	...	...	...	1	...	1	...	...
	...	...	1	...	...	...	...	...	...	...	1	1	...	1	1	...
	...	...	...	...	...	1	...	...	...	...	...	1	1	...	...	...
	1	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
	...	...	...	...	...	1	...	...	...	...	...	1	...	1	1	...
	...	...	...	1	...	1	...	...	...	...	1	1	...	...	...	...
	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
	..	...	1	...	...	...	...	...	...	...	...	...	...	...	...	...
	...	...	...	2	...	...	...	...	...	...	...	...	...	...	...	...
	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
	...	...	...	...	...	1	...	...	...	...	...	...	...	...	...	...

Table II. exhibits the distribution of the spot-groups in a table of double entry, the horizontal lines showing the numbers of groups yielding different synodic rotation periods, the vertical columns, the numbers of groups in each zone of latitude 5° wide.

This table brings out a most important point which has hitherto been practically entirely neglected, namely, the way in which the apparent spot movements in any particular zone of latitude differ among themselves. It will be seen that the differences of rotation period of the various groups in any particular zone of latitude are much greater than the differences of mean rotation between the different zones. The extreme differences shown in Table I. lie between 26·3 days and 29·0 days (synodic rotation periods); that is to say, within an extreme range of 2·7 days. The groups with periods between these limits have been printed in heavier type in Table II. If we turn to this latter table we see that in one particular zone of latitude, +10° to +15°, the extreme range is from 24·4 days to 31·2 days. In other words, if these spots persisted for nine consecutive rotations and travelled continuously at the same rate, the most quickly moving spot would make ten rotations and the most slowly moving only eight rotations, whilst the spots with mean period would be completing nine. The bearing of this fact upon the inquiry into the connexion between sun-spots and magnetic disturbances is obvious. It is not in the least necessary to presume that a disturbance indicating a long rotation period is due to a spot in high latitude. It is perfectly true that when we are dealing with the *mean* motions in any zone we find that the rotation period generally lengthens with the distance from the equator; but, as Table II. also shows, the individual spot-groups giving the longest periods are by no means situated in the highest latitudes.

A more detailed examination of the table shows that above 25° latitude, north or south, there is scarcely any tendency in the spot-groups to concentrate upon one particular period. The higher the latitude the more evenly are the groups scattered amongst the different rotation periods; a fact which has an important bearing upon the law of the variation of rotation rate with latitude. Carrington's expression for the rotation period involves the term $\sin^3 \lambda$, and Spoerer's the term $\sin (41° 13' + \lambda)$. The fractional exponent in the one case and the auxiliary angle in the other are refinements which have no warrant in the observations. It is perfectly clear that beyond 25° from the equator we can attach no great precision to the rotation period derived from spot-groups, since groups in higher latitudes are not only few in number, but appear almost accidental in the rotation periods which they yield; so that there is no justification for departing from a simple expression of the form $a - b \sin^2 \lambda$. The formula $875'{\cdot}7 \mp 164' \sin^2 \lambda$ satisfies the observation sufficiently well for all but the extreme latitudes, as will be seen by the eighth and ninth columns of Table I., which give

respectively the values of the daily angular movement as computed by this formula, and the differences of the observations from them.

The important discovery of Carrington of what has been described as "the systematic variation of rotation rate from equator to poles" has generally obscured this striking and remarkable variety in the motions of spots in any given latitude. It has been forgotten that, whatever the cause which produces this variation of rotation rate with latitude, the causes producing difference of rate within any given latitude are more effective still.

The question has been raised as to whether the mean rotation period of the Sun as derived from the spots varies from cycle to cycle, or in different parts of the progress of any one cycle. Table III. exhibits the daily sidereal movement for the different zones of latitude for the two cycles treated separately; it thus corresponds to Table I.; but to save space the two columns of mean synodic and mean sidereal rotation periods have been omitted. The comparison of the table emphasises the remark just made that there is no sufficient warrant for seeking a more complicated expression than one of the form $a - b \sin^2 \lambda$, the differences between the results given by the two cycles being often so considerable.

TABLE III.

Comparison of the Two Cycles.

Latitude.	Cycle I.			Cycle II.			I.–II.
	No. of Groups.	Weight.	Daily Sidereal Motion.	No. of Groups.	Weight.	Daily Sidereal Motion.	
°			′			′	′
+35	1	4	789·3	1	8	812·8	−23·5
32½	0	0	...	3	21	827·5	...
30	2	9	818·2	9	62	834·4	−16·2
27½	13	78	821·9	23	134	849·6	−27·7
25	24	147	822·8	42	253	848·3	−25·5
22½	50	295	844·3	65	408	854·3	−10·0
20	61	358	853·1	106	660	861·7	− 8·6
17½	58	335	864·0	130	839	864·9	− 0·9
15	101	634	861·7	140	953	867·9	− 6·2
12½	124	796	865·1	167	1138	868·1	− 3·0
10	93	580	871·5	153	1012	870·5	+ 1·0
7½	58	360	874·6	96	627	873·8	+ 0·8
5	36	228	880·1	63	401	875·3	+ 4·8
+ 2½	21	136	880·6	42	256	878·4	+ 2·2
0	19	113	868·9	22	143	880·5	−11·6
− 2½	35	224	874·1	35	229	874·7	− 0·6
5	65	413	870·1	83	544	872·9	− 2·8

Latitude.	Cycle I. No. of Groups.	Cycle I. Weight.	Cycle I. Daily Sidereal Motion.	Cycle II. No. of Groups.	Cycle II. Weight.	Cycle II. Daily Sidereal Motion.	I.–II.
− 7½°	110	708	867′·8	130	862	870′·7	− 2′·9
10	126	804	867·9	163	1070	868·9	− 1·0
12½	110	671	868·2	184	1196	866·1	+ 2·1
15	114	713	865·5	163	1027	862·7	+ 2·8
17½	104	662	862·0	148	911	860·2	+ 1·8
20	66	424	854·0	122	771	857·8	− 3·8
22½	38	239	854·0	72	457	851·8	+ 2·2
25	21	117	849·6	48	305	844·0	+ 5·6
27½	12	71	832·5	32	202	847·1	−14·6
30	7	46	829·9	17	104	843·4	−13·5
32½	2	11	844·2	7	39	818·6	+25·6
−35	1	6	843·4	2	6	791·8	+51·6

One most remarkable peculiarity is common to both cycles; and since it was brought out by Carrington's inquiry two cycles earlier than the first of these it is probable that it expresses a real peculiarity of the solar rotation. In spite of the great irregularity in the rotation periods given by the spots in any particular zone, there does appear to be a distinct tendency for the shortest mean period to be given, not at the equator, but slightly to the north of it. The curve given by the different rotation periods is not precisely symmetrical with respect to the equator, and, *on the whole*, there appears to be a tendency for the periods in the northern latitudes to lengthen more rapidly with distance from the equator than with those of the southern. The question as to any variation in the rotation period during the progress of a cycle is a particularly difficult one to answer satisfactorily, since there is a well-defined tendency for the spots to seek special latitudes at different parts of the cycle; and consequently, as at the beginning of a cycle most of the spots are in high latitudes, and at the end in low, the mean rotation periods tend to shorten as the cycle progresses, and it is difficult to ascertain whether this effect is wholly a function of Spoerer's "Law of Zones," or whether some further cause is also at work.

In Table IV. the two cycles under discussion have been divided into three portions; three years in each cycle have been considered as years of maximum, viz. 1882–4 in the first cycle, 1892–4 in the second cycle, and have been taken as the centre periods. The years preceding these are the years of increasing, those succeeding them of decreasing activity. The examination has been confined to the zone 10° to 22½° in each hemisphere, since the latitudes higher than this belt are scarcely represented at all during decrease, and the latitudes lower than it are in like manner scarcely represented during increase. The com-

parison shows that there is no clear evidence of anything like a systematic change in the rotation period during the progress of any particular cycle, and very little evidence, if any, of a change from one cycle to another.

TABLE IV.

Comparison of the Rotation Periods given by Different Cycles and Different Parts of a Cycle.

Hemi-sphere.	Cycle.	Phase of Cycle.	No. of Groups.	Apparent Mean Daily Drift.	Mean Synodic Rotation Period.	Daily Sidereal Motion.	Mean Sidereal Rotation Period.
				′	d	′	d
North	I.	Increase	73	+ 4·4	27·12	855·4	25·25
		Maximum	112	+13·7	26·81	864·8	24·97
		Decrease	34	+12·8	26·84	863·8	25·00
		Entire cycle	219	+10·6	26·92	861·7	25·07
	II.	Increase	51	+17·0	26·70	868·1	24·88
		Maximum	187	+15·2	26·76	866·2	24·93
		Decrease	98	+14·3	26·79	865·3	24·96
		Entire cycle	336	+15·2	26·76	866·2	24·93
South	I.	Increase	36	+13·1	26·83	864·1	24·99
		Maximum	126	+11·9	26·87	863·0	25·03
		Decrease	76	+10·2	26·93	861·3	25·08
		Entire cycle	238	+11·5	26·88	862·6	25·04
	II.	Increase	23	+11·5	26·89	862·5	25·04
		Maximum	204	+ 9·5	26·95	860·5	25·10
		Decrease	149	+14·7	26·78	865·8	24·95
		Entire cycle	376	+11·6	26·88	862·7	25·04
All spots	I.	Increase	109	+ 7·2	27·03	858·3	25·17
		Maximum	238	+12·8	26·84	863·8	25·00
		Decrease	110	+11·0	26·90	862·0	25·05
		Entire cycle	457	+11·0	26·90	862·0	25·05
	II.	Increase	74	+15·2	26·76	866·3	24· 3
		Maximum	391	+12·2	26·86	863·3	25·02
		Decrease	247	+14·5	26·78	865·5	24·95
		Entire cycle	712	+13·3	26·82	864·4	24·98
All spots	Both cycles	Increase	183	+10·6	26·92	861·7	25·07
		Maximum	629	+12·4	26·85	863·5	25·01
		Decrease	357	+13·4	26·82	864·5	24·98
		Entire cycle	1169	+12·5	26·85	863·6	25·01

The very slight diminution of the period with the progress of the cycle shown by the last section of Table IV. may be looked upon as purely accidental, for it will be observed that the northern spots in the second cycle and the southern spots in the first cycle showed quite as distinct a progression in the other direction. We may conclude, therefore, that there is no evidence of a change in the rotation period during the progress of the cycle other than that which results from the change in the distribution of the spots in latitude.

But if we compare together spots of different durations there does seem some distinct evidence of a systematic difference. Table V. brings this out. The spots are divided into three classes : those observed on six or seven days are regarded as short-lived spots ; those on eight, nine, or ten, as spots of medium duration ; and those of eleven, twelve, thirteen, and fourteen days, as long-lived. In order to complete the comparison we have added the recurring spots, and it will be noted that there is on the whole a distinct tendency for the short-lived spots to give a shorter rotation period than those of longer duration.

TABLE V.

Comparison of the Rotation Periods given by Spots of Different Durations.

Hemi-sphere.	Cycle.	Duration of Group.	No. of Groups.	Weight.	Apparent Mean Daily Drift.	Mean Synodic Rotation Period.	Daily Sidereal Motion.	Mean Sidereal Rotation Period.
					′	d	′	d
North	I.	Short	79	275	+ 9·5	26·95	860·5	25·10
		Medium	141	853	+15·4	26·75	866·5	24·93
		Long	105	876	+ 9·2	26·96	860·3	25·11
		Recurrent	99	3449	+ 7·8	27·01	858·8	25·15
	II.	Short	127	445	+23·0	26·50	874·0	24·71
		Medium	175	1061	+19·8	26·61	870·9	24·80
		Long	224	1917	+11·3	26·89	862·3	25·05
		Recurrent	133	4980	+ 5·8	27·07	856·9	25·21
South	I.	Short	107	376	+17·9	26·67	868·9	24·85
		Medium	156	944	+20·2	26·60	871·3	24·79
		Long	149	1272	+ 5·6	27·08	856·6	25·21
		Recurrent	125	8572	+ 6·6	27·05	875·7	25·18
	II.	Short	157	545	+21·5	26·55	872·6	24·75
		Medium	209	1260	+15·7	26·74	866·8	24·92
		Long	242	2091	+ 7·3	27·03	858·3	25·16
		Recurrent	167	11150	+ 5·1	27·10	856·1	25·23
Both	Both	Short	470	1641	+19·0	26·63	870·1	24·82
		Medium	681	4118	+17·8	26·67	868·8	24·85
		Long	720	6156	+ 8·5	26·98	859·5	25·13
		Recurrent	524	19722	+ 6·1	27·07	857·2	25·20

3. *Recurring Sun-spots.*

Table VI. presents for the recurring spot-groups the same statistics as were given in Table I. for the several groups considered independently at each apparition. It has not been thought worth while to give a table for these long-lived groups similar to Table II., but Table VII., corresponding to Table III., gives a comparison of the two cycles for these recurring groups. There is no doubt that these groups are much more free from the effect of accidental motions than the groups when considered separately in each apparition. The mean sidereal period given by them is exactly

25·2 days

and the formula which satisfies the variation of rotation period with latitude, and has been adopted in Table VI., is

$$866'\cdot6 \mp 128' \sin^2 \lambda$$

Table VI.

Rotation Periods from Recurrent Spots for Different Zones of Latitude.

Latitude.	No. of Groups.	Weight.	Apparent Mean Daily Drift.	Mean Synodic Rotation Period.	Daily Sidereal Motion.	Mean Sidereal Rotation Period.	Daily Sidereal Motion computed.	O−C.
°			′	d	′	d	′	′
+30	3	92	−23·9	28·12	827·1	26·12	834·6	−7·5
27½	10	364	−12·8	27·72	838·2	25·69	839·3	−1·1
25	19	704	− 6·8	27·51	844·2	25·59	843·7	+0·5
22½	38	1401	− 1·8	27·34	849·3	25·42	847·9	+1·4
20	54	1909	+ 3·3	27·16	854·4	25·28	851·6	+2·8
17½	57	1985	+ 7·0	27·04	858·0	25·18	855·0	+3·0
15	62	2143	+ 8·4	26·99	859·5	25·13	858·0	+1·5
12½	76	2904	+10·0	26·94	861·0	25·09	860·4	+0·6
10	65	2552	+11·0	26·90	862·1	25·06	862·9	−0·8
7½	32	1203	+14·8	26·78	865·8	24·95	864·4	+1·4
5	25	870	+18·2	26·66	869·2	24·86	865·6	+3·6
+ 2½	19	572	+17·0	26·70	868·1	24·88	866·4	+1·7
0	11	398	+15·1	26·76	866·2	24·94	866·6	−0·4
− 2½	21	693	+16·5	26·72	867·6	24·90	866·4	+1·2
5	38	1356	+13·1	26·83	864·2	24·99	865·6	−1·4
7½	65	2742	+10·7	26·91	861·7	25·06	864·4	−2·7
10	75	3249	+ 9·7	26·94	860·8	25·10	862·9	−2·1
12½	80	3198	+ 9·5	26·95	860·6	25·11	860·4	+0·2
15	79	3021	+ 8·5	26·98	859·6	25·13	858·0	+1·6
17½	68	2395	+ 4·5	27·12	855·6	25·24	855·0	+0·6

Latitude.	No. of Groups.	Weight.	Apparent Mean Daily Drift.	Mean Synodic Rotation Period.	Daily Sidereal Motion.	Mean Sidereal Rotation Period.	Daily Sidereal Motion computed.	O–C.
°			′	d	′	d	′	′
−20	58	2176	+ 0·5	27·26	851·6	25·37	851·6	0·0
22½	38	1664	− 4·8	27·44	846·3	25·53	847·9	−1·6
25	22	911	- 9·2	27·57	841·9	25·66	843·7	−1·8
27½	18	516	−10·4	27·64	840·6	25·70	839·3	+1·3
30	11	340	−15·7	27·84	835·4	25·86	834·6	+0·8
−32½	2	84	−24·4	28·13	826·7	26·13	829·7	−3·0
All spots ...		...	+ 6·1	27·07	857·2	25·20	...	...
Carrington		...	...	27·275	851·06	25·38	...	...

TABLE VII.

Comparison of the Two Cycles for Recurrent Spots.

Latitude.	Cycle I.	Cycle II.	I.–II.	Latitude.	Cycle I.	Cycle II.	I.–II.
+30°	823′·9	832′·5	− 8′·6	− 7½°	861′·0	862′·5	− 1′·5
27½	828·5	841·3	−12·8	10	858·6	863·0	− 4·4
25	838·3	845·8	− 7·5	12½	859·5	861·6	− 2·1
22½	845·0	853·7	− 8·7	15	861·4	858·0	+ 3·4
20	850·0	858·0	− 8·0	17½	857·5	853·6	+ 3·9
17½	857·9	858·1	− 0·2	20	854·7	849·6	+ 5·1
15	860·2	858·9	+ 1·3	22½	845·3	846·6	− 1·3
12½	864·7	858·9	+ 5·8	25	842·4	841·5	+ 0·9
10	865·2	860·4	+ 4·8	27½	835·5	844·8	+ 9·3
7½	865·3	866·2	− 0·9	30	829·2	838·9	+ 9·7
5	871·4	867·3	+ 4·1	−32½	...	826·7	...
+ 2½	869·6	866·3	+ 3·3	North spots	858·8	856·9	+ 1·9
0	864·7	870·2	− 5·5	South spots	857·7	856·1	+ 1·6
− 2½	867·8	867·4	+ 0·4				
5	864·3	864·1	+ 0·2	All spots	858·1	856·4	+ 1·7

4. *Summary.*

The Greenwich sun-spot measures for the two complete cycles 1879–1901 give us, therefore, the following results :

(1) Carrington's period does not correspond to a latitude of between 10° and 15°, as he supposed, but is given by the separate spot-groups of latitude 22½° ; by the recurrent spots of latitude 20°.

(2) Carrington's period is not that of the mean of all spots, but is considerably longer than that mean.

(3) A different rotation period is given if the examination is restricted to the movements of each appearance of the groups considered separately and to those of the spots of long duration.

The latter—the recurrent spots—give a somewhat longer period in the mean, and are more accordant *inter se* than are the groups treated separately.

(4) The curve given by the different rotation periods is not precisely symmetrical with respect to the equator, the zone of shortest rotation being north of the equator.

(5) The rotation periods given by different spots in the same zone of latitude differ more widely than do the mean rotation periods for different zones of latitude.

(6) Spots of short duration tend to give a shorter rotation period than spots of long.

(7) There is no evidence of a progressive change in the mean rotation period during the progress of a sun-spot cycle other than that which follows from the gradual shift of sun-spot activity from higher to lower latitudes.

(8) A comparison of the rotation periods from the separate groups for the two cycles shows an apparent slight shortening of the period for the northern hemisphere whilst the southern is absolutely unchanged. It cannot be presumed that this apparent change in the northern hemisphere is anything but accidental.

(9) For when the recurrent spots are taken there is a slight retardation of the rotation period from the first cycle to the second, shown by both northern and southern hemispheres.

86 *Tyrwhitt Road, St. John's, S.E.:*
1905 *June* 9.

Observations of Mars, 1903. By Major P. B. Molesworth, R.E.

The apparition of 1902–3 was rather an unfavourable one, as the diameter of the planet at opposition was only 14″·57. The great tilt, however, of the axis presented the northern regions in most favourable position for observation, and the small diameter of the polar cap permitted the details to be followed up nearly to the pole. The southern regions of course were greatly foreshortened.

Vernal equinox, N. hemisphere	1902 August 12
Aphelion of *Mars*	1903 January 13
Summer solstice, N. hemisphere	February 27
Mars in opposition	March 29
Autumnal equinox, N. hemisphere	August 28

The north latitude of the centre of the disc decreased from +22°·6 on January 17, when the observations began, to +21°·09 on February 2, increasing again to +25°·9 on June 2, after

which it again decreased—very rapidly towards the end—to +11° on September 21, when the last observation was made.

The place of observation was Trincomali, Ceylon. Longitude $5^h\ 24^m\ 55^s{\cdot}6$ E. and latitude 8° 33′ 24″·2 N. A 12¾-inch Calver equatorial reflector was used throughout, and a power of 450 was almost always employed. Regular work on the planet was carried on from February 13 to June 7 with three interruptions, due to my absence from Trincomali—March 3–20, April 7–14, and May 8–19. A few fragmentary observations were also made somewhat earlier and later, the first date being January 17, the last September 21. Table I. gives the particulars of work each month.

TABLE I.

Résumé of the Observations.

Month. 1903.	Total Nights.	Number of Nights Available.	Observations made. Number of Nights.	Observations made. Percentage of Total.	Observations made. Percentage of Available.	Total Number of Hours.	Average Hours per Night.	No. of Drawings made.	Remarks.
Jan.	14	...	1	...	...	2·00	2·00	3	
Feb.	12	...	2	...	...	4·17	2·08	4	
	16	16	15	93·7	93·7	25·66	1·71	22	Period of regular observations
Mar.	31	15	15	48·4	100·0	23·00	1·53	14	
Apr.	30	24	21	70·0	87·5	43·91	2·09	31	
May	31	29	13	41·0	44·8	25·58	1·97	19	
June	7	7	6	85·7	85·7	8·33	1·39	5	
	23	...	4	...	...	5·17	1·29	3	
July	31	...	2	...	...	1·83	·91	1	
Aug.	31	...	...	...	...	...	...	...	
Sept.	21	...	4	...	...	2·33	·58	2	
Total for period of regular observations	115	91	70	60·9	76·7	126·48	1·81	91	
Grand total ...	247	...	83	...	...	139·98	1·68	104	

During the period of regular observation, February 13–June 7, the weather was, as a rule, abnormally fine, and the observing conditions were very often perfect. The air was recorded as being perfectly transparent on 77·2 per cent. of the occasions of observation, and normally clear on 16·6 per cent. There was perfect steadiness, due to the absence of wind, on 99·6 per cent.; and the definition was recorded as "good" to "perfectly sharp" in the same proportion, being recorded as "perfect" or "sharp" on 70·2 per cent. A magnifying power of over 400 was used in 77·5 per cent. of the observations.

I made 104 drawings in all during the year—forty-nine on the larger blanks and fifty-five on the smaller. Ten drawings were also made with the same telescope by Lieutenant J. S.

Barker, R.E., and two rough sketches by my brother, Mr. G. N. L. Molesworth. Of these drawings I forward copies of six of the most characteristic, which may be regarded as showing the typical appearance of the various aspects of *Mars* in 1903.

I carefully measured the mean coordinates of a number of the most prominent markings upon a large number of the sketches. To effect this a large disc of *Mars* was drawn and the lines of latitude and longitude inserted with the proper tilt of the axis. This diagram was then reduced by photography to the exact size of the disc blanks used for the drawings. Transparencies were then made by contact on lantern plates, and purposely very lightly developed, so as to leave the white absolutely clear glass. The glass positive was placed over each drawing and correctly oriented as far as possible. The coordinates of the required markings within 30° of longitude of the central meridian were then read off and corrected for the longitude and latitude of the centre of the disc at the time of the drawing. In addition to the above, central meridian transits of various points were taken by eye near the opposition. The values obtained by both methods and the resultant coordinates adopted are given in Table II. (Table V. in the manuscript).

TABLE II.

Coordinates of Points.

Point.	Longitude. Measured from Drawings.	No.	From C.M. Transits.	No.	Adopted Value.	Latitude measured from Drawings.	Remarks.
	°		°		°	°	
Gihon estuary ...	5·4	9	6·1	3	5·5	− 5·5	
Siloë Fons	5·9	7	9·1	1	6·5	+30·3	
Thymiamata (centre)	...	...	10·8	1	...	...	
Oxia Palus	16·0	9	22·9	1	18·0	+13·0	
Mare Acidalium (lake)	18·5	17	18·4	2	18·5	+47·0	
„ „	24·2	16	...	...	24·2	+35·0	
Margaritifer Sinus ...	22·3	13	33·0	2	26·0	− 5·0	
Niliacus L. (E. end)...	28·5	11	...	...	28·5	+27·5	
Mare Acidalium (lake)	28·9	12	33·3	1	29·5	+56·2	
Prom. Aromatum ...	...	...	40·7	2	...	...	
Mare Acidalium (lake)	41·5	11	...	...	41·5	+31·7	
Niliacus L. (W. end) .	46·5	12	...	...	46·5	+24·0	
Mare Acidalium (lake f.)	52·8	14	...	...	52·8	+30·1	
Auroræ Sinus ...	54·8	11	59·7	1	55·5	−10·3	
"Tanais Lacus" ...	63·0	14	...	...	63·0	+52·2	
Nectar estuary ..	62·4	7	64·6	1	63·0	−28·7	
Lunæ L. (centre) ...	65·7	11	...	...	65·7	+17·5	
L. Hyperboreus (centre)	68·2	9	...	...	68·2	+77·4	

Point.	Longitude. Measured from Drawings.	No.	From C.M. Transits.	No.	Adopted Value.	Latitude measured from Drawings.	Remarks.
Tithonius L.... ...	82°·0	9	81°·7	4	81°·8	−14°·3	
Lacus Solis (centre) .	87·2	7	84·9	5	86·0	−31·1	
Mareotis L. (E. end) .	95·0	10	... ...		95·0	+31·9	
Lacus Phœnicis ...	97·1	8	... ...		97·1	−18·8	
Junction of Tanaïs and Ceraunius W. ...	... ...		99·2	1	...	...	
Ascræus L.	104·8	6	... ...		104·8	+ 8·3	
Centre white spot (Arcadia)	... ...		112·5	2	112·5	...	
Arsia Sylva	120·0	2	114·0	2	114·0	− 5·0	
Mareotis L. (W. end)	118·3	12	121·0	2	119·0	+24·0	
Bocca Sirenum ...	121·7	7	130·6	2	125·0	−20·0	
Mœotis Palus ...	128·8	10	123·0	2	127·0	+52·9	
Nodus Gordii ...	135·5	3	136·4	1	133·0	+ 6·2	Ad. lat. + 9·0
Phrygius Lacus ...	147·0	11	... ...		149·0	+ 7·0	„ +12·0
Castorius L. (E. end)	152·0	4	... ...		152·0	+45·7	
Euxinus L.	158·1	12	159·3	4	158·5	+38·7	
Arsenius L.	162·0	2	... ...		...	+60·0	
White polar spot F. (E. end)	162·8	5	... ...		162·8	+77·0	
Titanum Sinus ...	166·0	9	166·2	1	166·0	−19·2	
Ammonium	167·0	7	... ...		167·0	+14·7	
Erebi Fons	169·6	5	... ...		169·6	+26·6	
Propontis	176·4	9	185·1	1	176·5	+39·1	
Trivium Charontis ...	200·0	6	205·4	3	201·0	+15·1	
Stygia Palus... ...	203·6	4	... ...		203·6	+26·0	
Laestrygonum Sinus .	203·7	4	... ...		203·7	−15·4	
Pambotis L.	221·1	7	225·7	2	222·0	+ 6·6	
Morpheos L.... ...	222·4	5	... ...		222·4	+31·6	
Cyclopum Sinus ...	225·0	6	... ...		225 0	−13·3	
White polar spot F. (W. end)	241·8	3	... ...		241·8	+76·8	
M. Cimmerium (W. end)	253·5	6	248·5	(2)	251·5	+ 4·8	
Nubis L. (centre) ...	254·0	10	251·8	(2)	253·0	+27·1	
Oniri Palus	260·1	7	... ...		260·0	+39·1	
Syrtis Minor... ...	267·3	6	... ...		267·3	− 0·8	
"Casius L"	268·6	7	... ...		268·6	+46·5	
Mœris L.	281·3	9	... ...		281·3	+12·2	

Point.	Longitude. Measured from Drawings.	No.	From C.M. Transits.	No.	Adopted Value.	Latitude measured from Drawings.	Remarks.
	°		°		°	°	
Nilus L.	283·8	5	... ...		283·8	+35·4	
Copaïs L.	285·0	9	278·9	1	284·0	+56·1	
N. end Syrtis Major .	286·1	10	288·4	1	286·5	+27·6	
White circular spot ...	287·6	5	... ...		287·6	+ 7·8	
Major Lacus... ...	293·9	6	... ...		293·9	+ 9·3	
"Astaboras L." ...	295·7	12	... ...		295·7	+20·3	
Hellas (centre) ...	296·0	2	... ...		296·0	−37·0	
Coloë Palus	296·1	11	... ...		296·1	+42·2	
Hellas (N. end) ...	297·0	3	... ...		297·0	−24·3	
"Hippus L."... ...	297·8	6	... ...		297·8	− 9·0	
Hipponis L.	308·4	10	... ...		308·4	+ 3·7	
Hammonis Cornu ...	313·7	11	... ...		313·7	−10·6	
"Anubis L."... ...	320·5	1	... ...		...	+21·0	
Arethusa L.	328·1	11	... ...		328·1	+56·2	
Ismenius L.	329·9	13	337·0	2	332·0	+37·8	
Sigeia Portus ...	337·2	10	337·6	3	337·3	− 8·3	
Edom (centre) ...	...	...	348·4	1	...	...	
"Ortygia Fons" ...	351·5	2	... ...		348·0	+54·0	
Hiddekel estuary ...	355·5	12	0·2	1	356·0	− 6·0	
Fastigium Aryn ...	358·5	2	... ...		358·5	− 6·0	

(*Part II. follows here in the manuscript, and gives a detailed description of the various markings on the planet in order, the planet being divided for the purpose into eight regions; the belt lying within 60° of the equator on either side being divided into six regions, each about 60° of longitude in breadth, whilst the areas within 30° of the South Pole and of the North Pole form the seventh and eighth regions respectively. Table III., given below* [*Table VI. in the manuscript*], *shows the number of drawings available for the study of each region. The six drawings selected for reproduction represent the chief features of the six divisions of the equatorial belt, together with the north polar region. The south polar region was practically invisible during the whole of the apparition. Key maps are also added, in which Schiaparelli's nomenclature, with the amendments of the latest Mars Report of the British Astronomical Association, has been followed throughout, except in part of the Syrtis Major, where Lowell's nomenclature appeared more suited to the configurations of the year under discussion. Here and there Major Molesworth has given a provisional name of his own to features which appear to be new or hitherto unnamed.*—The Secretaries.)

TABLE III.

Sect.	Limits of Longitude.	Limits of Latitude.	Breadth.	Region.	No. of Drawings. M. L.	M. S.	B. S.	G.M. S.	Total.
1	310° to 10°	+60° to −60°	60°	Sinus Sabæus	11	10	4	1	26
2	10 „ 70	+60 „ −60	60	Mare Acidalium	10	11	2	0	23
3	70 „ 120	+60 „ −60	50	Lacus Solis	9	13	...	...	22
4	120 „ 180	+60 „ −60	60	Mare Sirenum	8	9	1	...	18
5	180 „ 250	+60 „ −60	70	Mare Cimmerium	3	8	1	...	12
6	250 „ 310	+60 „ −60	60	Syrtis Major	7	5	2	...	14
7	0 „ 360	−60 „ −90	360	S. Polar Region	...	...	...	...	...
8	0 „ 360	+60 „ +90	360	N. Polar Region	48	56	10	1	115

In the above table the letters M., B, and G.M. at the head of the columns are the initials of the three observers; the letters L. and S. refer to the sizes of the prepared discs upon which the drawings were made, L. denoting the larger discs and S. the smaller.

The six drawings selected for reproduction are the following:

TABLE IV.

Fig.	No of Sketch.	Region.	G.M.T.	d	h	m	Longitude. °
1	72	Sinus Sabæus and Mare Acidalium	April	30	1	51	9·0
2	69	Mare Acidalium and Lacus Solis ...	„	27	4	17	71·54
3	59	Mare Sirenum	„	19	4	27	145·07
4	51	Mare Cimmerium	„	14	5	13	200·10
5	49	Mare Cimmerium and Syrtis Major	„	7	4	13	246·90
6	45	Syrtis Major	„	2	4	0	287·27

TABLE V.

Index of Names of Martian Details.

1. Pyrrha.
2. Deucalion.
3. Sinus Sabæus.
4. Furca.
5. Edom.
6. Thymiamata.
7. Margaritifer Sinus.
8. Chryse.
9. Typhon.
10. Euphrates.
11. Hiddekel.
12. Gihon.
13. Oxus.
14. Indus.
15. Jamuna.
16. Protonilus.
17. Deuteronilus.
18. Niliacus L.
19. Lunæ L.
20. Dioscuria.
21. Arnon.
22. Cydonia.
23. Acidalium M.
24. Tempe.
25. Ortygia.
26. Tanais.
27. Thaumasia.
28. Nectar.
29. Solis L.
30. Tithonius L.
31. Phœnicis L.
32. Ganges.
33. Chrysorrhoas.

KEY MAP

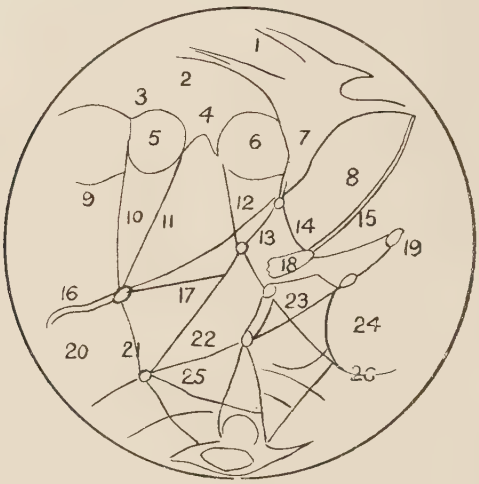


Fig. 1.

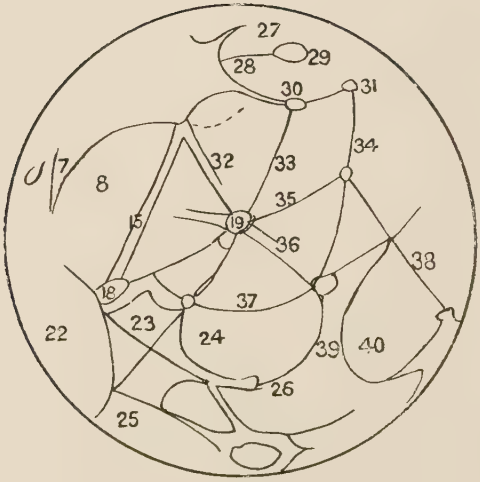


Fig. 2.

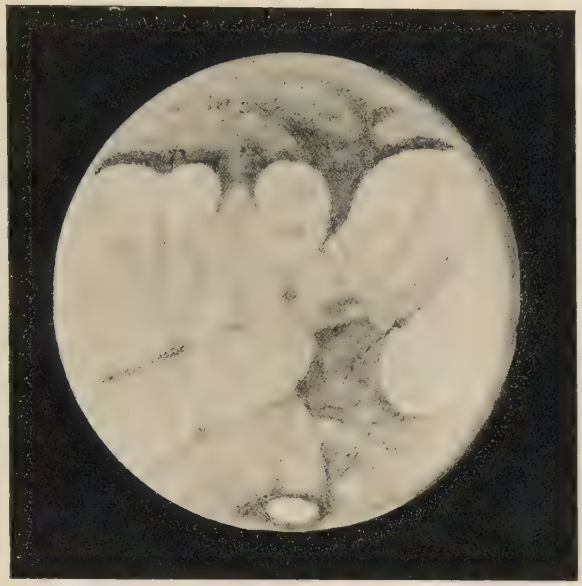


Fig. 1.—1903 April 30^{d} 1^{h} 51^{m} G.M.T.
Reference No. 72. Powers, 375, 450.
$\lambda = 9^{\circ}\cdot08$; $\phi = +24^{\circ}\cdot7$; $a = 13''\cdot11$.

Fig. 2.—1903 April 27^{d} 4^{h} 17^{m} G.M.T.
Reference No. 69. Powers, 375–700.
$\lambda = 71^{\circ}\cdot54$; $\phi = +24^{\circ}\cdot6$; $a = 13''\cdot37$.

KEY MAP.

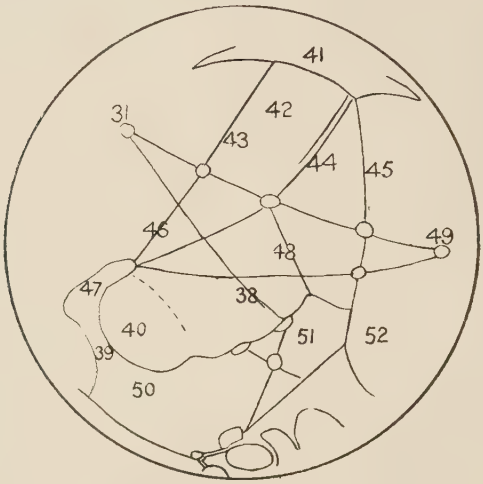


Fig. 3.

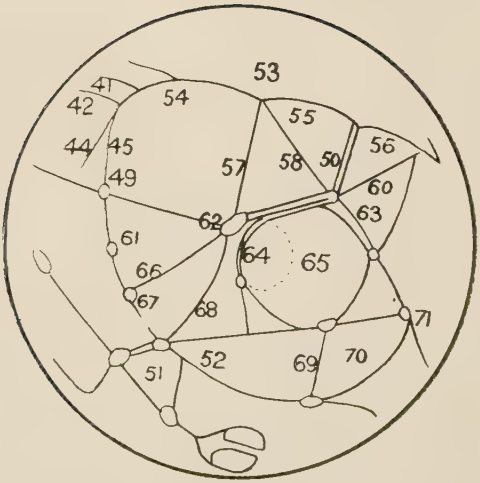


Fig. 4.

Fig. 3.—1903 APRIL 19^{d} 4^{h} 27^{m} G.M.T.
Reference No. 59. Power 450.
$\lambda = 145°{\cdot}07$; $\phi = +24°{\cdot}01$; $a = 13''{\cdot}97$.

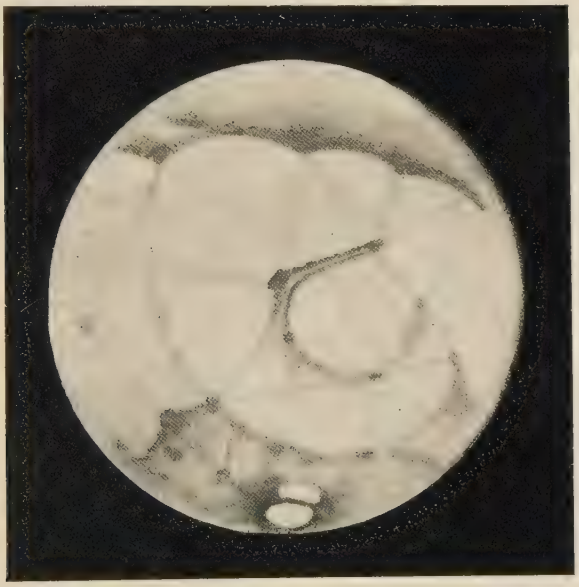


Fig. 4.—1903 APRIL 14^{d} 5^{h} 13^{m} G.M.T.
Reference No. 51. Power 450.
$\lambda = 200°{\cdot}10$; $\phi = +23°{\cdot}8$; $a = 14''{\cdot}27$.

KEY MAP.

Fig. 5.

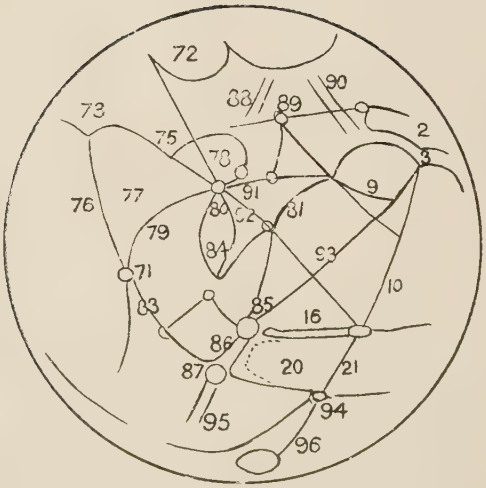


Fig. 6

Fig. 5.—1903 APRIL 7^d 4^h 13^m G.M.T.
Reference No. 49. Power 450.
$\lambda = 246°{\cdot}90$; $\phi = +23°{\cdot}4$; $a = 14''{\cdot}48$.

Fig. 6.—1903 APRIL 2^d 4^h 0^m G.M.T.
Reference No. 45. Power 450–700.
$\lambda = 287°{\cdot}27$; $\phi = +22°{\cdot}9$; $a = 14''{\cdot}56$.

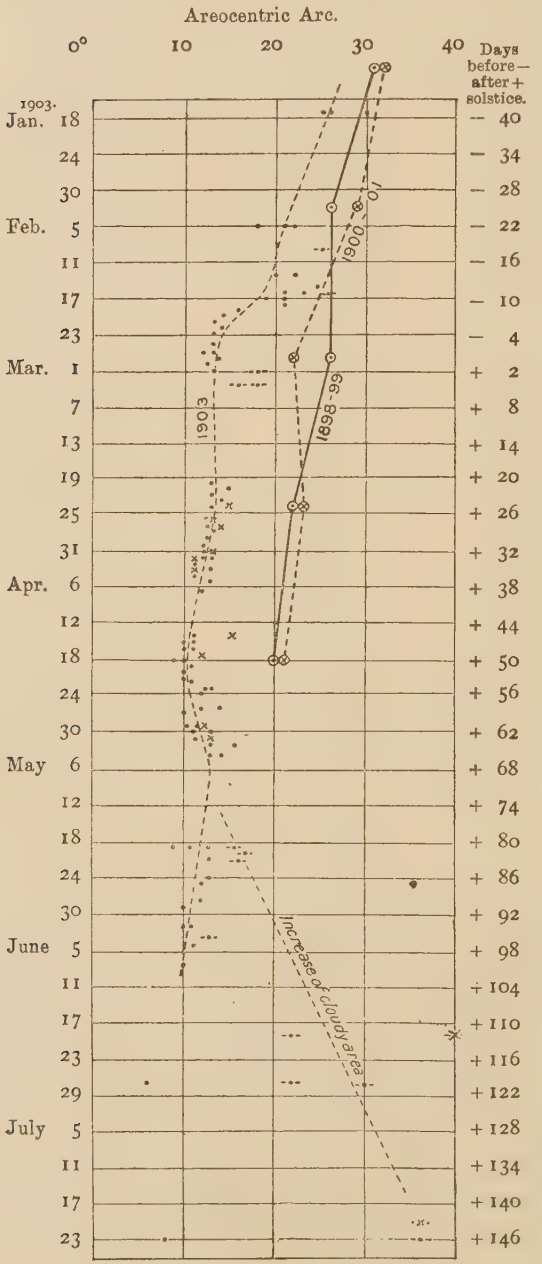
34. Iris.
35. Uranius.
36. Nilus.
37. Dardanus.
38. Phlegethon.
39. Ceraunius.
40. Arcadia.
41. Sirenum M
42. Memnonia.
43. Arduenna.
44. Gigas.
45. Titan.
46. Macrobrius.
47. Mareotis L.
48. Lycaon.
49. Ammonium.
50. Nerigos.
51. Titania.
52. Cebrenia.
53. Cimmerium M.
54. Zephyria.
55. Acolis.
56. Cyclopia.
57. Læstrygon.
58. Antæus.
59. Cyclops.
60. Cerberus.
61. Titanum L.
62. Trivium Charontis.
63. Eunostos.
64. Styx.
65. Elysium.
66. Erebus.
67. Erebi Fons.
68. Hades.
69. Anian.
70. Ætheria.
71. Nubis L.
72. Ausonia.
73. Tyrrhenum M.
74. Hesperia.
75. Syrtis Minor.
76. Adamas.
77. Lybia.
78. Abyssinia.
79. Thoth.
80. Mœris L.
81. Casuentus.
82. Alcyonius.
83. Casius.
84. Nilotis.
85. Coloë Palus.
86. Boreosyrtis.
87. Copais L.
88. Lunæ Pons.
89. Hippus L.
90. Solis Pons.
91. Bubastis.
92. Pallas.
93. Phison.
94. Ismenius L.
95. Pyramus.
96. Kison.

The six drawings (Plates 21–23) illustrate the chief features of the six divisions of the equatorial and temperate zones of the planet during the apparition of 1903; the following section deals with the chief changes noted in the north polar region.

North Polar Region.

The north polar regions were most favourably placed for observation in 1903, the decrease of the polar caps permitting of observation quite close to the pole, while the north latitude of the centre of the disc was practically at a maximum. In Table VI. (Table VII. in manuscript) I have given the areocentric arc subtended by the polar cap as measured off each of the drawings; the diagram gives the same information graphically; and on it I have also plotted the arc subtended by the polar cap at the same Martian season in previous apparitions. No deviation of the centre of the cap from the pole of rotation was detected.

With the melting of the snows white detached spots appeared in the polar shades. The largest, whitest, and most pronounced of these spots was shown first rather indefinitely on February 5. It was steadily seen throughout April, and was very


Areocentric Arc.
0° 10 20 30 40
Days before— after + solstice.
1903.
Jan. 18 — 40
24 — 34
30 — 28
Feb. 5 — 22
11 — 16
17 — 10
23 — 4
Mar. 1 + 2
7 + 8
13 + 14
19 + 20
25 + 26
31 + 32
Apr. 6 + 38
12 + 44
18 + 50
24 + 56
30 + 62
May 6 + 68
12 + 74
18 + 80
24 + 86
30 + 92
June 5 + 98
11 + 104
17 + 110
23 + 116
29 + 122
July 5 + 128
11 + 134
17 + 140
23 + 146
1903
1898-99
1900
Increase of cloudy area

distinct in May and early in June, after which it appeared to become merged in the haze which overspread the region. As a rule it was an elongated, curved spot, shaped rather like a carraway seed, concentric with the pole, and at times almost as brilliant as the polar cap. It was separated from the latter by a very dark, narrow, curved line, and also from *Panchaia* by a very dark streak. In addition to the detached spots a white star-disc-like brilliant circular spot was very distinctly seen at the border of the cap on April 27, 29, 30, and May 2.

The history of the polar cap in 1903 may be divided into four stages :

First Stage. From the commencement of the observations to about the middle of February. Cap, bright white, decreasing rapidly ; bordered by marshes with no very definite boundary.

Second Stage. From the middle of February to about the first week in June. Cap, intensely white, decreasing very gradually, sharply bounded by an intensely black line shading into the marshes. During this stage the details near the cap were seen with great distinctness ; and several detached white spots joined in the shades surrounding it.

Third Stage. From about June 10 to end of July. Disintegration of the cap and formation of a fog-like envelope of considerable extent, sometimes allowing the minute bright nucleus to be dimly seen. The surrounding dark line had either ceased to exist, or become merged in the haze.

Fourth Stage. Disappearance of the cap, which was replaced by a large indefinite very dull lightening of fog or mist, with no definite boundaries. The date of this change cannot be fixed exactly owing to the paucity of the later observations, but was probably some time early in August. This stage appears to have been accompanied by a general fading of the Martian maria. Comparing the behaviour of the polar cap this year with the phenomena in 1898 and 1901, it is apparent that the decrease in area was much more rapid this year, and its diameter at the summer solstice much smaller than in previous years. After the solstice, however, the decrease was exceedingly gradual, and the cap survived with great brilliancy for some time.

In the following table (Table VI.) the areocentric arc subtended by the polar cap on each of the drawings is given. In this table D = the number of days before (−) or after (+) the summer solstice of the northern hemisphere, λ° the longitude of C.M. of the centre of the disc, and a the areocentric arc subtended by the cap.

Figures in ordinary type are those determined from my own drawings ; figures in heavy type from drawings by Lieut. Barker (B.) and G. Molesworth, Esq. (G.M.).

In the diagram each dot represents the arc measured from one of my sketches ; each cross a measure from Lieut. Barker or G. Molesworth, Esq. Where the cap was indefinite I give three dots, the centre one darkest (thus . ● .).

TABLE VI.

Areocentric Arc subtended by N. Polar Cap.

Date. 1903.	D.	λ°.	a.	Remarks.
Jan. 17	−41	346	30	} 27°.
	−41	356	26	
	−41	9	25	
Feb. 5	−22	170	21	} 20°·5.
	−22	179	22	
	−22	188	18	
9	−18	145	25 ±	Rather indefinite
13	−14	106	22	} Rather indefinite, ± 22°.
	−14	115	20	
15	−12	79	25	
16	−11	72	23	} 22°.
	−11	83	21	
	−11	90	± 26	Indef.
17	−10	63	21	} 20°.
	−10	75	19	
18	− 9	52	21	
19	− 8	44	16	} 16°.
	− 8	57	16	
20	− 7	31	14	
21	− 6	25	13	} 13°
	− 6	36	13	
22	− 5	21	12	Small. 14°?
23	− 4	12	13	
24	− 3	2	15	
25	− 2	353	13	
26	− 1	339	12	} 12°·5.
	− 1	352	13	
27	± 0	336	13·5	Solstice.
28	+ 1	327	12·5	
Mar. 1	+ 2	319	13	
3	+ 4	289	18	} Indef. ± 17°.
	+ 4	302	16	
20	+21	41	13	
21	+22	46	15	
22	+23	30	13	Exceedingly white.
23	+24	21	14	
Mar. 24	+25	9	13	
	+25	**15**	**15**	**B.**
26	+27	350	12·5	
	+27	**356**	**13**	**B.**
27	+28	345	12·5	
	+28	**353**	**14**	**B.**
28	+29	334	13	
29	+30	327	12·5	Opposition.
30	+31	315	12	
31	+32	307	12	
	+32	**320**	**13**	**B.**
Apr. 1	+33	298	12	Fading.
	+33	298	13	
	+33	**306**	**11**	**B.**
2	+34	287	11	Marshes dark
3	+35	280	13	
	+35	**289**	**11**	**B.**
4	+36	275	11	
5	+37	269	13	
7	+39	247	12	
14	+46	187	11	
	+46	**193**	**15**	**B.**
	+46	200	10	
15	+47	139	11	
	+47	178	10	
16	+48	139	11	
	+48	169	10	
17	+49	121	12	
	+49	**165**	**12**	**B.**
18	+50	112	9	
	+50	155	10	
19	+51	145	11	
20	+52	90	10	
21	+53	82	10	
	+53	124	10	
22	+54	63	11	
23	+55	76	12·5	

Date. 1903.	D.	λ°.	*a.*	Remarks.
ate.				
)03.				
r. 23	+55	110	13	
24	+56	101	12	
26	+58	84	14	
	+58	94	12	
27	+59	71	10	Very white spot at S. end
29	+61	6	11·5	
	+61	23	12	B.
	+61	51	10·5	Very white spot S.E.
30	+62	9	13	
	+62	42	11	Very white spot S. central.
y 1	+63	349	13	
	+63	0	13	B.
	+63	33	11	
2	+64	26	13	White spot S. central.
	+64	36	16	
4	+66	332	14	Diffuse.
	+66	8	13	
19	+81	182	9	Diffuse, faded.
	+81	190	11	Not very dark.
	+81	235	13	
	+81	247	16	Diffuse.
20	+82	220	17	,,
21	+83	162	13	Sharp

Date. 1903.	D.	λ°.	*a.*	Remarks.
May 21	+83	202	13	Sharp.
24	+86	188	13	,,
25	+87	135	12	,,
28	+90	132	12	Fairly sharp.
29	+91	95	10	Sharp.
	+91	133	10	Marshes very dark.
	+91	143	10	
June 1	+94	78	11	,,
	+94	106	10	Fairly dark.
3	+96	94	13	Rather diffuse
4	+97	45	11	Fairly sharp.
7	+100	20	10	Edge diffuse.
19	+112	297	22	Faint edge very nebulous.
	+112	301	40	**Very diffuse (G M.)**
27	+120	184	22	Very faint.
	+120	218	30	White central spot ± 6°.
July 20	+143	329	36	**Very diffuse (G.M.)**
23	+146	294	36	White centre ± 8°.
Sept. 14	+199	132	...	Gone.
17	+202	99	...	,,

Special Features.

Geminations.—Very few cases of gemination have been seen during the apparition. The following are the most distinct cases.

Boreosyrtis.—Certainly double, April 2–5.

Casius.—This marking is certainly not a canal in the true sense of the word, but merely a darker knotted edge to shaded Utopia. It was certainly anomalously double in the early part of April.

Cerberus.—This marking appears to be really allied to the narrow *maria*. Certainly double on April 14.

Nilosyrtis (*Nilotus*).—Certainly double April 4 and 5. Components very close.

Pierius.—Double at beginning of April.

Protonilus.—Very dark, knotted, and irregular. I cannot regard Protonilus this year as a true canal. In spite of the negative observation of March 1 and April 2, I am inclined to

believe that it was double for a considerable portion of the apparition, but the irregularity of the components made the doubling very difficult to see.

Pyramus.—Doubtfully double on April 4.

I have not considered in the foregoing the anomalous duplications of *Nilokeras* and *Ceraunius*. These objects are coarse, and consist obviously of two distinct streaks, and are in no sense true double canals. *Nilus*, *Ganges*, and *Jamuna* were all broad with a certain slight appearance of gemination. All the above geminations were best visible in the early part of April. This may perhaps be due simply to the excellent observing conditions and the size of the disc, then at its greatest.

Projections.—The following cases of projection were seen in 1903 :—

January $17^d\ 12^h\ 04^m$ G.M.T. A very white spot, like a projection, at the limb, position angle ± 205. (Optical? Argyre.)

February $22^d\ 12^h\ 20^m$ G.M.T. (Power, 600). "Suspicions of a very small projection from the terminator close to its S. end ($\lambda^\circ = \pm 308^\circ$ ϕ -25° to -30°). The impression was persistent for a short time, but before I could make certain of it the sky became too bright." (Doubtful.)

March $3^d\ 12^h\ 35^m$ G.M.T. (375). N. end of Hellas appearing to project outside curve of terminator.

April $16^d\ 4^h\ 35^m$ G.M.T. (450). Distinct, very small, brilliant projection on N.W. limb, not far from the polar cap. Exact position marked on No. 55. Visible for about five minutes in the form of a small brilliant circular spot appearing to project as a minute white mound beyond the limb. (Real.)

May $4^d\ 4^h\ 51^m$ G.M.T. Slight deformation of terminator just N. of Lunæ L. marked on No. 79. Terminator rather brighter at this point than elsewhere. (Certainly real.)

July $23^d\ 2^h\ 20^m$ G.M.T. Slight apparent bulge on the terminator just S. of the line Proto-Deuteronilus. (Certainly optical. Arabia.)

Twilight at the Terminator.—On several occasions during this apparition I have been much struck with the breadth and intensity of the twilight shading along the terminator. It seems to me this is much greater than is permissible by theory, and would lead us to expect an atmosphere of greater density on *Mars* than is usually accepted. On the other hand the distinctness of the darker markings close to the terminator and the comparative absence of atmosphere effects on *Mars* militate against such a conclusion. The real explanation may be that the constitution of the Martian atmosphere differs entirely from that of the Earth.

General Remarks.

The Importance of Good Air.—I am more than ever convinced of the importance of good air and perfect definition in

the study of Martian detail. On some of the glorious nights in March and April this year the appearance of *Mars* is absolutely different from what it is with the same power and seemingly sharp (but not quite perfect) definition. Markings which seem straight uniform streaks under the latter conditions break up into most complex structures under perfect definition, and details can be seen which were hitherto unsuspected. Yet the change in the definition is very slight indeed, merely the difference between "sharp" and "very sharp"; and the untrained eye would scarcely detect it.

These perfect nights are rare, even in Ceylon; but I am satisfied that experience of one or two of them would convince the most sceptical of the objective reality of the majority of the "so-called canals."

Classification of the Details.—I think it is high time that the detail on *Mars* was specialised and classified in something the same way as that of the Moon has been.

At present the details visible on *Mars* may be divided into: (i.) the polar caps; (ii.) the polar "marshes"; (iii.) the maria or "seas"; (iv.) the "continents"; (v.) the "half-tones"; (vi.) the "canals"; (vii.) the oasis or "lakes."

Of these subdivisions (iii.), (iv.), and (v.) grade insensibly into each other, with no rigid line of demarcation. We have every variety of tone from brilliant white areas, such as Argyre and Edom, through half-tones like Hesperia and Deucalion, and heavily shaded areas like Baltia and Utopia, barely distinguishable from the lighter maria, to the almost black tones in the Syrtis Major, Sinus Sabæus, and Mare Acidalium.

Under the heading of "canals" also we have (as pointed out by the Rev. T. E. R. Phillips in *Monthly Notices*, vol. xliv. No. 1, p. 40) several utterly distinct phenomena "lumped together" under one generic name. Large knotted streaks, shaded areas of considerable extent with darker edges, broad uniform smudges, irregular edges to dark areas, slight changes of tone, narrow well-defined streaks, and fine delicate lines at the limit of vision are all impartially claimed as "canals." This is, to say the least of it, unscientific. It should be possible (though I admit not easy) to classify these features in some more satisfactory and scientific manner, and such a classification would do much to simplify the work of investigation. I give a rough idea of the sort of classification I would propose. In this the detail is grouped into the main divisions: (A) continents, (B) maria, (C) "canals" (for want of a better word). The lakes and half-tones occurring in the continents I would group under (B), and the half-tones in the maria under (A). The subdivisions would be somewhat as follows:—

(A) *Continents.*—(1) Bright continental areas partially bordered by very dark maria (*e.g.* Edom, Chryse).

(2) Continental areas bordered by faint edgings, generally representing a slight change of tone (*e.g.* Amazonis, Eden).

(3) Areas bordered by distinct streaks with or without a change of tone (*e.g.* Dioscuria, Cydonia).
(4) Shaded areas bordered by distinct streaks (*e.g.* Titania, Utopia).
(5) Half-tones, generally more or less rectangular in outline, occurring in the maria (*e.g.* Hesperia, Atlantis).
(6) Islands, generally circular or oval in outline, occurring in the maria (*e.g.* Hellas, Argyre, Thyle).
(7) Faint shaded areas with no definite streak edge (*e.g.* northern portion of Thaumasia).

(B) *Maria.*—(1) Large variegated maria of irregular outline (*e.g.* Syrtis Major).
(2 Elongated maria, more or less rectangular in outline, sometimes seen double (*e.g.* M. Cimmerium, Sinus Sabæus).
(3) Detached large lakes, roughly circular or oval in outline (*e.g.* L. Solis).
(4) Detached smaller lakes, roughly circular or oval (*e.g.* Siloë Fons).
(5) Large detached irregular lakes (*e.g.* M. Acidalium).
(6) Small irregular lakes (*e.g.* Oxia Palus [?]).
(7) Half-tones, with darker edgings, occurring in the continents (*e.g.* Ceraunius, Nilokeras).

(C) "*Canals.*"—(1) Large irregular streaks, sometimes double (*e.g.* Cerberus, Protonilus).
(2) Broad diffuse streaks, sometimes double (*e.g.* Jamuna, Gigas).
(3) Narrow uniform streaks (Læstrygon, Gihon).
(4) Narrow uniform lines (the Lowell interpretation of the canals).
(5) Irregular darker edges to half-tones (*e.g.* Casius, Granicus).
(6) Streak edges to half-tones (*e.g.* Pierius, Deuteronilus).
(7) Edges to half-tones, with no definite darker streak (*e.g.* Poros, Cantabias).

Some such classification as this would immensely simplify the consideration of the markings on *Mars*. In many cases the different classes would merge insensibly into each other, in which case I would suggest giving the two subdivisions connected by a hyphen (*e.g.* Cerberus might be classified as B_2-C_1).

The Gemination of the Canals. — The gemination of the canals of *Mars* does not, after all, appear to be such a miraculous phenomenon. I must admit that I do not understand a narrow uniform line with the "oases" on it suddenly becoming widely double ; the companions lying some distance on either side of the position of the single canal, and each being a faithful replica of the original with all its uniformity and all its knots. Is this, however, a fair account of the phenomenon of gemination as it usually appears to most eyes ?

Speaking for myself, I would say "Certainly not." The cases

of gemination I have seen on *Mars* have usually occurred in subdivisions C_1 and C_2 of the above classification. In the first of these classes the phenomenon, as already pointed out by Maunder in 1892, is strongly analogous to the lightenings which appear in the centres of the narrower maria (class B_2) (*Mars Report*, B.A.A., 1892, p. 197). In the second class the "canal" under all but the best definition appears a broad uniform streak; but, when the seeing is at its best, this streak is found to be bounded along either side by slightly darker lines. There is no sudden shift in position of either component, and the gemination generally occurs in the coarser, not the finer markings. I cannot see in this any tendency of the canals to "jump with magical fissiparity at distances of 300, 400, or 500 miles." Such phenomena may occur, but I have never seen them in the course of several years' careful study of *Mars*.

A peculiarity of *Mars*, which has been little noticed, may throw light on the question of gemination. I refer to the strong parallelism which undoubtedly exists in many regions between different canals. Several such parallel sets can easily be mentioned: *e.g.* Iris, Sirenium, Titan, Læstrygon, Cyclops, Amenthes; Araxes, Gorgon, Brontes; Tartarus, Antæus, and other instances too numerous to mention.

Two such parallel canals with a slight included shade, if close together, would at once give the appearance of gemination. If only one canal, or the faint included shade alone, was seen, the canal would certainly be noted as single. This would explain the apparent anomaly of a canal being seen narrow and single, broad and single, and nearly doubled by different observers at the same time. In the first case only one of the bounding canals would be seen, the fainter one and the included shade being missed; in the second only the included shade with no darker edgings is seen; whilst in the third the included shade would be missed and only the darker edgings detected.

Reality of the Canals.—Personally, I am quite convinced of the reality of the great majority of the so-called canals; I think I could have convinced the most sceptical on this point if they could only have spent an hour or two at my telescope on some of the perfect nights in March and April this year. I do not mean to say that the fainter ones exist exactly in the form in which we see them. They are, to quote Mr. Maunder, "the integration of markings far too small to be separately defined." As in previous years, even the fainter ones appear to me to have evidence of structure. They are "streaky" not linear, the "streaky" appearance being most distinct when the definition is best. They are more like a streak made on very rough paper with a round-pointed crayon or stump than an ink-line drawn with a pen. The Læstrygon is, I think, the most uniformly linear of them.

Detail in the Maria.—This year I have had the opportunity of carefully studying two of the larger maria (the Mare Acidalium

and the Syrtis Major) under practically perfect conditions. From the results of this study I am convinced that the nature of the delicate detail is the same all over the planet. The edges to the maria are as a rule canaliform objects, of the classes C_5, C_6, and C_7. There are in the maria the same delicate streaks; similar to, and in continuation of, those in the continents, with darker lakes at the points of intersection. The only difference is in the tone of the background on which the details are seen. Naturally they are more easily detected on a light background than on a dark one. The canaliform edges of the maria and the "canals" traversing them are almost always in prolongation of the continental "canals," and are obviously connected phenomena.

Illusion and Contrast.—Experiments by Maunder, Lane, and others in the last few years with artificial discs have suggested the possibility that the canals are in some cases illusions. These experiments have been interpreted by some as clearly proving that all the canals are illusions; but this conclusion is not warranted by the experiments. Wherever a non-existent canal is drawn, there are always (to quote Mr. Maunder) "a few minute markings much too small to be seen individually," a crease in the paper, or a slight change of tone. In other words, the detail is there, but beyond the resolving power of the eye, which consequently translates it into the simplest form.

But where does reality end and illusion begin? It is absurd to attribute markings like Cerberus, Ganges, Nilosyrtis, and Agathodæmon to illusion. They are as incontestably real as the belts of *Jupiter*, and when well placed are ridiculously easy objects on anything like a good night, even with comparatively small apertures. Is it not reasonable to suppose that the employment of larger apertures (within limits) and higher powers, in good air, will reveal similar but more delicate details quite as real as these; and that careful study of the planet will add to their number? In this case the illusion theory need only be invoked to explain comparatively few cases out of a very large number.

The contrast theory brought forward lately by M. Antoniadi is plausible, and may be held to explain some of the delicate details on *Mars*. But if pressed to extremes it becomes a most dangerous argument. If we carry it to its logical conclusion we shall have a *Mars* deprived of all "nuances" of light and shade, both light and dark markings being broad masses of uniform tone unrelieved by any half-tones or delicate shading. But its most dangerous tendency is in exalting hurried sketches of *Mars* to a higher level of accuracy than drawings based on careful and extended study of the planet, as, according to the contrast theory, these conditions tend to strain the eyes and produce illusion.

Up to date, all experienced planetary observers have insisted on the importance of prolonged and persistent study. If we carry the contrast theory to excess, however, all this is wasted time. The careful series of drawings made under such conditions

will be so vitiated by eye fatigue and the effects of contrast as to be useless as contributions to our knowledge of the planet.

The expert observer is far better off than the tyro in one respect. He will have much less difficulty in discriminating between reality and illusion.

The illusive doublings, for instance, produced by inferior definition, wrong focus, or fatigue of the eye are so patent to the expert, that they lose all power to deceive. This necessary experience can, however, only be gained by prolonged study.

If the majority of the delicate details on *Mars* are simply due to contrast, the same explanation should hold good with many coarser details on the other planets.

Let us apply the contrast theory to the detail on *Jupiter*, which presents some points of resemblance to that on *Mars*. We might say that the darker edgings of the equatorial belts, the brighter spots bordering them, the longitudinal white rifts in these belts, and the duplicity of the fainter belts are all producible by contrast. Possibly they are, but it does not follow that they are so produced. On the other hand most of these phenomena are incontestably real, and are even shown on some of the imperfect photographs which have been made of the planet.

Are we then justified in assuming their reality in the case of *Jupiter*, while we ascribe the very similar phenomena seen on *Mars* entirely to contrast?

If we could use a power of 3000 or 4000 on *Mars* efficiently, I think we should find that many of the details ascribed to illusion or contrast are realities. The larger image would enable us to break up the detail into recognisable forms. The geometrical and linear forms which the markings of *Mars* assume to many eyes merely result from the minuteness of the component details, and the inability of the eye to grasp their real nature.

NOTE.—*The report of his observations of Mars in* 1903 *presented to the Society by Major P. B. Molesworth, from which the preceding paper has been extracted, was too long for reproduction in full in the "Monthly Notices." The Council felt, however, that no mere abstract would satisfactorily set forth the great amount of work on the details of the planet which the report represented, and decided, therefore, to place the complete manuscript in the Library, where it will be at all times available for consultation and reference, and to print only the chief tables and general conclusions which Major Molesworth has given.*—THE SECRETARIES.

On the Relative Efficiency of Different Methods of Determining Longitudes on Jupiter. By A. Stanley Williams.

The proof of Professor G. W. Hough's further paper on this subject has been read with much interest by me, as it defines his position more clearly with regard to several questions at issue. My remarks on his paper will be considered under four heads—namely, (1) The apparent or accidental errors of the observations ; (2) Systematic error ; (3) The theoretical side of the question ; and (4) Application to the planet *Saturn*.

(1) *The Apparent Errors of the Observations.*

My first paper on this subject, published in the *Monthly Notices*, 1904 March, was entirely confined to the question of the apparent or accidental errors of observation ; all consideration of systematic error, which includes both "personal equation" and what has been termed "variable error," having been purposely excluded. And it has now, I should imagine, been firmly and incontrovertibly established that when a large mass of observations by the micrometric method are compared with a correspondingly large series by the method of transits in exactly the same manner, the apparent errors of the observations are for all practical purposes the same with either method. With both methods the average mean error of an observation is $\pm 2^{m}\cdot 0$ for spots in general, this value being reducible to $\pm 1^{m}\cdot 5$, or less, under exceptionally favourable circumstances.*

There is not very much in Professor Hough's present paper bearing directly on this subject. He states that he finds the mean error of Schmidt's observations to be $\pm 2^{m}\cdot 65$ when they are reduced with a variable increasing rotation period. It is necessary to point out, however, that this result still cannot be accepted as comparable with any of the similar data contained in my writings, for the reason that the variable increasing rotation period is not the one satisfying Schmidt's observations, but that derived from Professor Hough's own measures.

And apparently this is considered to be a fair way of comparing the results obtained by the two observers ! I propose to return to this manner of deriving the apparent errors of observation later on. It should also be remarked that the existence of Schmidt's "constant error," varying with the planet's hour angle, has not in any way been disproved. All that has been done is to show that, as I had anticipated, the apparent errors of Schmidt's

* To my mind it is much more important to know what are the apparent errors of the observations for spots in general than it is for those made under exceptionally favourable circumstances. The mean error for the five spots with a "large number of observations" (see *Monthly Notices*, vol. lxv. p. 171) observed by Professor Hough with the micrometer is $\pm 2^{m}\cdot 5$, which is distinctly larger than the $\pm 2^{m}\cdot 0$ found for the mean error of an observation by the method of transits for the same class of spots.

uncorrected observations are considerably lessened when the latter are reduced with a variable increasing rotation period, even with one not satisfying the observations in question in the best possible manner.

But in forming an opinion as to the accordance of Schmidt's observations it is important not to overlook his earlier results. His twelve observations of a dark spot in 1862 show a mean error of $\pm 0^{m}\cdot 6$; his six observations of a sharp elbow in the same year one of $\pm 3^{m}\cdot 0$; and his five observations of a white spot in 1866 one of $\pm 1^{m}\cdot 2$. The mean of these three results is $\pm 1^{m}\cdot 6$, which is practically the same as that given by the best micrometer observations, and considerably smaller than that shown by his latest work, at any rate as regards the uncorrected observations. And this leads to the question whether there may not be some reason for the apparent errors of these latest observations being perhaps a little large should his "constant error" on a proper discussion prove to have no existence. No one can possibly have a more profound respect and admiration for Schmidt and his work than the writer. Nevertheless it must be remembered that these latest observations were made when he was seventy-five years of age, and the work is one in which quickness of perception probably plays an important part. It has been necessary to go into this matter of Schmidt's observations more deeply than I should have wished, because Professor Hough apparently pins his faith solely on the later observations of this illustrious observer alone. But surely this is not reasonable now that we have so much additional work by many other well-known skilful observers available for purposes of comparison!

Dr. O. Lohse states, I think,* that the probable error of his central meridian transits is $\pm 2^{m}$, although these, as has already been pointed out, were sometimes made with the help of a micrometer and sometimes without, and therefore do not form a perfectly homogeneous series of observations. It should be repeated that the mean error of Schmidt's corrected observations is only $\pm 1^{m}\cdot 5$. The following additional data may be added for comparison with those contained in the list on pp. 433, 434 of the *Monthly Notices* for 1904 March. Nine observations of the red spot by Mr. W. F. Denning in 1883 show a mean error of $\pm 2^{m}\cdot 3$ (*Astronomical Register*, vol. xxi. p. 172).† Ten observations by the same observer in 1904-5 October to March give one of $\pm 1^{m}\cdot 2$ (*Observatory*, 1905, p. 188). Fifteen observations of the red spot hollow by the Rev. T. E. R. Phillips in 1904 give a mean error of $\pm 1^{m}\cdot 5$ (private letter). Nine observations of the middle of the red spot in 1903–4 by the

* I have mislaid the part of the Potsdam Observatory *Publications* containing his results, so that I am unable to verify this figure or to work out the actual mean errors of his observations.

† Really rather less, for the ephemeris does not satisfy the observations well.

writer show a mean error of $\pm 1^m \cdot 5$; and ten of the following end show one of $\pm 2^m \cdot 1$ (*A. N.* 3983). Twenty-five observations (not yet published) of the middle of the red spot by the writer during the past opposition give a mean error $\pm 1^m \cdot 5$. The mean of these six additional results is $\pm 1^m \cdot 7$, practically the same as that found before.

The real rotation period of a spot is never known, and the only fair way in which we can form an idea of the accidental errors entering into a determination is to compare the observations of any particular observer with an ephemeris satisfying the motion of the spot *according to the observations of that observer*. In almost all astronomical problems we are confronted with the same difficulty, and I believe I am right in saying that it is the universal custom to derive the mean or probable errors in any particular investigation from the accordance of the observations *inter se*. It must be remembered that we are here dealing only with the apparent errors of observation, not with the systematic errors. Yet Professor Hough has derived the apparent errors of the observations made by one observer by comparing them with an ephemeris derived from the observations of another observer. That is, he takes an ephemeris which necessarily satisfies in the best possible way his own observations. A method more advantageous to the micrometrical method and more obviously unfair to the method of transits it would be difficult to imagine.

(2) *Systematic Error.*

That the micrometric method is free from all kinds of systematic error has most certainly never been proved. A few no doubt accidental instances of more or less close agreement with the Marth-Crommelin ephemeris, or with this ephemeris "corrected to conform to the *true* * rotation period," cannot be held to prove it. It would not be difficult to adduce cases of equally satisfactory agreement on behalf of the method of transits. For example, Professor E. E. Barnard's observations of the red spot in 1891 (*Monthly Notices*, vol. lii. p. 12) agree excellently with the ephemeris of Marth—far better, in fact, than do the micrometer observations of 1887 in the instance referred to by Professor Hough. On the other hand, there is no difficulty in selecting cases where the micrometer observations show, from internal evidence, clear signs of being subject to systematic error. Professor Hough admits that he has to "correct" the Marth-Crommelin ephemeris before his observations (of the red spot?) can be made to conform to the same; and he also admits that he not infrequently has to use a variable rotation period, sometimes even one with three terms. But in making these admissions it seems to me that he simply cuts the ground from under his feet, for by similarly "correcting"

* *I.e.* the rotation period satisfying Professor Hough's observations

the Marth-Crommelin or any other ephemeris,* or by making use of a suitable variable rotation period, the majority of the errors that have been designated as "variable errors" or "cumulative errors" can without difficulty be accounted for or corrected. Hence these admissions really prove that the micrometric method is itself liable to the same systematic errors! Certainly the variations thus corrected in the micrometer observations are in several cases exactly of the same apparent character as those to which the method of transits is subject, and which have been referred to as "variable errors" or "cumulative errors."

Several instances of large variable or systematic errors affecting the micrometric method have been already pointed out. As another example, I would now invite serious attention to the following figures, given in the "Report" of the Director of the Dearborn Observatory for 1885, p. 13, as the variable rotation period deduced for the red spot in 1884–5 :—

				h	m	s
1884 Sept.	25 to Dec.	3	Rot. per. =	9	55	44·0
Dec.	3 „ Feb.	2		9	55	40·1
1885 Feb.	2 „ April	4		9	55	39·1
April	4 „ May	15		9	55	38·7
May	15 „ June	29		9	55	42·0

Since we read that the motion of the spots on *Jupiter* is "smooth, never abrupt," and that abrupt motion is denied, above all in the red spot, it is evident that the remarkable differences here shown cannot be regarded as real. Hence, to quote Professor Hough's own words, slightly modified : † "Here we have apparently a well-established fluctuation in the rotation period of 5·3 seconds (shown by the micrometer measures) which did not exist"! As the red spot is stated to have become more conspicuous, "so much so as to be readily seen with moderate optical power," it is clear that the foregoing remarkable fluctuations cannot be ascribed to the faintness of the spot.

The following statement seems open to argument. "The objects that are observed are usually many millions of miles in area, and presumably have mass. We should not expect any abrupt change in direction or mode of motion in a moving mass." But what reasons have we for presuming that the spots in general have any (considerable) mass? The inference rather is, it seems to me, that the surface density of the planet is probably slight. Moreover, it does not necessarily follow that a given change is due to an actual transference of matter. Many of the known changes on *Jupiter* are possibly only apparent, and attributable rather to the effects produced by the condensation and expansion of vapours and gases, to convection currents, or to comparatively slight changes of brightness or reflecting power. It is

* Including that which satisfies the micrometer observations.

† *Monthly Notices*, vol. lxiv. p. 828.

well known, too, that very sudden and very considerable changes do sometimes occur on *Jupiter*.

Professor Hough's explanation of his discordant observations of a white spot in 1881 hardly seems to me sufficient; but assuming it to be so, is not this explanation a tacit admission of the validity of my contention as to one way in which "variable error" may arise? "The apparent displacement was simply due to the different reference point in making the measures." This is just what I have been endeavouring to point out is a frequent source of origination of systematic error with the method of transits.*

The position with regard to the whole matter of systematic error may be summed up as follows. The published micrometer observations and results possess ample internal evidence showing that they are subject to most if not all of the systematic errors to which the method of transits is liable; besides being, perhaps, also subject to other errors peculiar to the use of the micrometer. At present we have nothing definite to guide us as to the relative size of these errors, nor can we have, so far as I can see, until we possess comparable contemporaneous series of observations with the micrometer by several different observers, such observers working quite independently at distant stations, and using telescopes ranging in aperture from, say, 6 inches up to 18 inches. Professor Hough is therefore not justified in treating all, or even the larger part, of the differences found to exist between his micrometer observations and those of other observers, made by the method of transits, as being wholly due to the latter method.

(3) *Theoretical.*

This side of the question has not, I think, been treated quite fairly by Professor Hough. It is no very difficult matter to compute the shift in seconds of arc of a point near the centre of a planet's disc due to rotation in a given interval of time. For the present purpose use has been made of a convenient formula published by Professor F. L. O. Wadsworth in the *Observatory*, 1897, p. 369, viz.

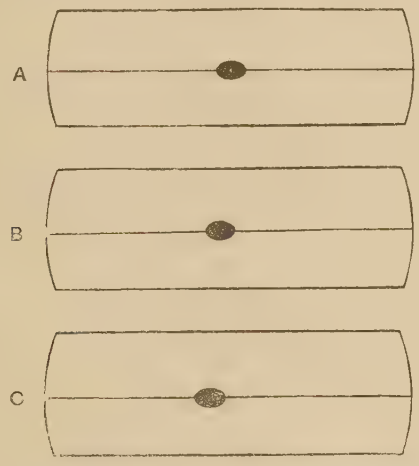
$$\epsilon = \frac{21600}{M} \cdot 0^{\cdot}00029 \cdot \delta$$

where ϵ is the angular shift in seconds of arc of a point near the centre of a planet's disc per minute, M is the time (in minutes) of axial rotation, and δ the apparent angular semi-diameter (in seconds of arc) of the planet.

It is necessary to decide upon the diameter of the planet to be adopted. The equatorial diameter of *Jupiter* may vary during the observable period from $50''$ to about $30''$. The diameter we ought to adopt, however, is that diameter which corresponds to the distance at which the average of the whole number of

* See *Monthly Notices*, vol. lxv. p. 177.

observations is made. Probably at least three-fourths of all observations are made within the period comprised between two and three months preceding opposition and the two or three months following the same. Likewise the great majority of all spots observed are situate within 20° of the planet's equator, so that the reduction in scale due to latitude will, on the average, be small. Consequently 40″ would seem to be a fair diameter to adopt as that at which the observations are made. For short intervals of time, such as five to ten minutes, the shift of a spot near the planet's central meridian due to rotation may be assumed to occur at a uniform rate. The rotation period of the planet we may take to be $9^h\ 55^m$. Using, then, the above data the average shift of a spot near *Jupiter's* central meridian in one minute of time will be 0″·21 ; in five minutes it will be 1″·05 ; and in ten minutes 2″·1.



In order to show the effect of this displacement, the three accompanying figures have been drawn as carefully as possible to scale. They represent the rotational displacement of a spot on *Jupiter* three seconds in length. Fig. A shows the spot in mid-transit, fig. B when it is five minutes past transit, and fig. C when it is ten minutes past transit. I believe that most of those who will carefully examine these diagrams will agree with me in concluding : (1) that an experienced observer should be able to observe the transits with an average mean error of $\pm 2^m{\cdot}0$; (2) that under very favourable circumstances such an experienced observer should be able to observe the transits with an average mean error of $\pm 1^m{\cdot}5$, or even less ; (3) that an apparent shift or error of central meridian amounting to as much as five minutes is altogether out of the question, much more so is one of ten minutes or more, as we are asked to believe (it is

hardly likely that it could amount to more than a minute); (4) that, provided the spot is well defined, there is no reason why the disc should not be bisected by it with as much accuracy as by a micrometer wire. If the spot were ill defined or irregular in shape, then the measures made of it would naturally suffer in the same degree as the bisection of the disc by the spot itself.*

As corollary to the foregoing it may be concluded that, since an apparent shift or error of central meridian amounting to much more than a minute is inadmissible, the known displacements coming under the description of "variable error," "cumulative error," "personal equation," and "variable personal equation," cannot be accounted for by any such hypothesis. In other words this means, it seems to me, that the cause of the errors in question, or at any rate the larger part of them, must necessarily be ascribed to something else, namely, to the different manner in which different observers observe the same planetary markings.† Consequently micrometer measures by different observers would necessarily be subject to all and the same kinds of systematic error. And, further, it follows that all the statements and figures, so far as they are correct, contained in Professor Hough's papers respecting the amount and the nature of the systematic errors affecting the method of transits will apply, and in all probability with like force, to the micrometric method. That this is the right way of regarding the matter is shown clearly by Professor Hough's statement that the "fictitious central meridian chosen for any group seems to be accidental, since for different spots observed on the same night the central meridian is different." It would be difficult to find a more convincing proof that the systematic errors in question are not ascribable to the choice of a "fictitious central meridian," but are inherent in or connected with the appearance of the spots themselves. It is also easy to imagine the conditions under which the "cumulative error" may

* The analogy drawn by Professor Hough between the transit of a Jovian spot and that of a star over an imaginary wire bisecting the field of a transit instrument, from which the real wires are supposed to be removed, hardly seems to the writer to be a happy one, because the distance to be bisected would be many times larger in the latter case than in the former. A better illustration would be to separate the wires of a micrometer by 40″ and observe the time when a star is exactly in mid-transit between the two. I should imagine that an observer, after a little practice, could observe such times with a very high degree of accuracy. The only advantage the micrometric method would seem to have is the power of repetition; but this appears to be counterbalanced by the uncertainty in setting on the limbs, and the prejudicial effect caused by placing a wire over a spot.

† Or the same observer at various times and under different circumstances. I fancy that practically everyone who has ever undertaken the work of comparing and discussing observations and drawings, not merely of *Jupiter*, but those of other planets, will be in agreement with the writer as to the sufficiency of this cause to produce the known variable and other systematic errors. It must not be forgotten that the appearance of a spot is *largely* dependent both on the size of the telescope used and on the state of the seeing, quite apart from any question as to the personal idiosyncrasies of the observer.

arise, but it is almost inconceivable that an observer could unconsciously select a "fictitious" central meridian for a particular spot differing progressively and in a regular manner with the time from the true central meridian, and yet not applying to neighbouring spots observed on the same nights. But we have still more positive proof, if such were required, that these variable and other systematic errors cannot possibly be due to the selection of a "fictitious" central meridian. For when the transits are observed with the help of a micrometer wire, set to half the measured diameter of the disc, these errors occur just the same.

(4) *Application to Saturn.*

It has been shown that the average angular shift of a point near the central meridian of *Jupiter's* disc per minute is $0''\cdot21$; and also that the average mean error of the micrometer observations is $\pm 2^m$, corresponding to a shift of $0''\cdot42$. If we assume the equatorial diameter of *Saturn* to be $18''\cdot6$, and the time of rotation to be $10^h\ 30^m$, we shall get $0''\cdot092$ as the shift per minute of a point on the planet's equator. The spots observed on *Saturn* in 1903 were in a rather high latitude, however. The measures for latitude appear to be somewhat discordant, but probably we shall not be far out if we take it to be $+36°$. The ellipticity of the disc complicates the matter, but multiplying $0''\cdot092$ by the cosine of the latitude we shall get $0''\cdot074$ as the corresponding shift per minute of a spot near the central meridian of *Saturn's* disc in latitude 36°, $\pm 2^m\cdot0$, and this will be quite near enough. Since the average mean error expressed in arc of the micrometer measures on *Jupiter* is $\pm 0''\cdot42$, a shift of equal amount on *Saturn* in latitude 36° will correspond to 5·7 minutes of time in observing the transit. This, then, is the average mean error that Professor Hough's measures of the white spots on *Saturn* might be expected to have. Under very favourable circumstances a somewhat smaller value might be anticipated, but it will hardly be seriously contended, I should imagine, that the spots on *Saturn* were as well suited for measurement as the great red spot on *Jupiter* was at the time of its greatest plainness. It may be taken, therefore, that $5^m\cdot7$ is a minimum value, and it might be nearly twice as great, judging from the micrometer measures of certain Jovian spots. If, therefore, the few micrometer observations of the spots on *Saturn* show a mean error much smaller than $\pm 5^m\cdot7$, such accordance must assuredly be fictitious. This is the case, and such small mean residuals as $\pm 2^m\cdot3$ and $\pm 0^m\cdot8$ are certainly fictitious, as likewise must be any conclusion regarding the variable motion of one of the spots, based on the scanty given data. It may be mentioned that the mean error of an observation by the method of transits of the principal spot from twenty-eight observations discussed by Professor H. C. Wilson * is $\pm 7^m\cdot8$. This is somewhat larger than the theoretical value,

* *Popular Astronomy*, 1903, p. 445.

but this might be expected, seeing that the observations were made by quite a large number of observers, and the spot was certainly more difficult to observe than the average Jovian spot. Mr. Denning has amply explained the reason for some of the larger discordances in the *Monthly Notices*, vol. lxiv. p. 242.

It would be interesting to apply these and some other considerations to the planet *Mars*, and to the case of a fictitious planet of the same size and at the same distance as *Jupiter*, but rotating in half the time that the latter does; but this must be deferred.

Hove: 1905 *June* 5

Addendum to "Discussion of the Greenwich Observations of the Satellite of Neptune" in Monthly Notices, Vol. LXV., pp. 570–583, by Messrs. Dyson and Edney.

1. It should have been stated that the quantities $s \sin dp$ and ds dealt with in this paper are in the sense "Tabular—Observed," and the resulting values of da, dN, &c., are subtracted from the tabular places to give the results of the observations.

2. On p. 581, although the result is not affected, it would have been more correct to compare the values of N and I found at Greenwich for 1903·1 with the actually observed values found by Dr. Struve for 1890·3, instead of with the values found from the interpolation formula he derived from a discussion of his own and previous observations. The figures are

H. Struve ...	1890·4	N = 185°·27	I = 119°·16	
Greenwich ...	1903·1	N = 187·58	I = 117·40	

In 12·7 years dN = +2°·31, dI = −1°·76; and the annual changes of N and I are

$$d\text{N} = +0°·182 \text{ and } d\text{I} = -·°138 \text{ for } 1896·7.$$

Dr. Struve's result for 1874·0 being

$$d\text{N} = +0°·148 \text{ and } d\text{I} = -0°·165.$$

For the mean date 1896·7

$$\psi_2 = 40°·9 \text{ and } \sin \gamma d\theta = °·212$$

giving

1896·7 N = 186°·44 I = 118°·28 $\psi_2 = 40°·9$ $\sin \gamma d\theta = °·212$

to compare with Dr. Struve's result,

1874·0 N = 182°·78 I = 121°·99 $\psi_1 = 52°·6$ $\sin \gamma d\theta = °·208$

The close agreement of the determinations of sin $\gamma d\theta$ is satisfactory.

The figures on p. 581 should be replaced by

$$N_1M_1 = 52^\circ\cdot6 \qquad N_2M_2 = 40^\circ\cdot9 \pm 2^\circ\cdot5$$
$$M_1N_1N_2 = 121\cdot99 \qquad M_2N_2N_1 = 61\cdot72$$

and $$N_1N_2 = 3^\circ\cdot66$$

Solving the triangles, the values found for the inclination, &c., are the same as those previously given.

3. The inclination of *Neptune's* equator to the plane of its orbit derived from these figures is about 29°.

The Meteors from Biela's Comet. By W. F. Denning.

Undoubtedly the rich shower of *Andromedids* visible in the light of the nearly full moon on 1904 November 21 formed the most important meteoric event of the past year. The only observer of it in the United Kingdom appears to have been the Rev. W. F. A. Ellison, of Enniscorthy, who at $7^h\ 25^m$ G.M.T. saw eight meteors in fifteen seconds, and twenty-four altogether between $7^h\ 25^m$ and $8^h\ 25^m$. Twenty-two others were observed between $8^h\ 25^m$ and $9^h\ 25^m$, after which the numbers "fell off greatly." The radiant by eye estimation from forty or fifty tracks was at 21° + 50°. The meteors generally were very brilliant, with trains, and a few of the more conspicuous objects were recorded as under :—

1905.	G.M.T. h m		From ° °	To ° °
Nov. 21	8 2	>1	308 + 47	280 + 39½
„	8 49	♀	Low in W.	Andromedid
„	9 8	♃	337 + 7	329 − 7
„	9 16	♃	354 + 30	348 + 18
Nov. 26	7 35	♀	52 + 27	64 + 8½
28	8 50	>♀	215 + 50	215 + 46

The display apparently continued until November 28.

It was also observed by K. Bohlin, of Stockholm (*Ast. Nach.* 3997), who says that the radiant of twenty-eight meteors (the paths of which he gives in his paper) recorded on 1904 November 21, 5^h to 11^h (mid-European time) was found by the method of least squares to be about 3° distant from γ *Andromedæ* at

$$26^\circ\ 2' + 44^\circ\ 10'\ (1900)$$

The meteors were of considerable brilliancy. The first

indications of the display were noticed on November 16, and it appeared to have ceased on November 22.

The occurrence of these *Andromedids* (sufficiently brilliant and abundant to attract special notice in strong moonlight) is remarkable, as there was a rich shower of them in 1899 November 24, so the interval is only five years, and the inference is that the meteors are being rapidly spread out along a considerable arc of the orbit.

The observed perihelia of Biela's Comet occurred 1772 February 19 and 1852 September 23, so that the mean period derived from twelve returns is 6·71 years.

The great meteoric showers of 1798 December 7 and 1885 November 27 comprise thirteen periods of 6·69 years.

The periodic time of the comet in 1846 was 6·617 years (Hind), and slightly greater than this according to the computed (but unverified) returns to perihelion in 1859 May 23, 1866 January 25–26, and 1872 October 6. By adopting a mean period of 6·68 years the observations of the meteoric displays, &c., seem fairly well satisfied.

P.P. of Biela. Period of 6·68 years.	Observed Meteoric Display.	Earth's Long. ☌
1852 September 23	...	...
1859 May 30	...	...
1866 February 2	...	...
1872 October 9	1872 November 27	66·1
1879 June 14	...	...
1886 February 17	1885 November 27	65·8
1892 October 23	1892 „ 23	62·3
1899 June 28	1899 „ 24	62·3–6
1906 March 3	1904 „ 21	59·1

There will probably be a storm of these *Andromedids* in 1905 November 17–21 ; this, if realised, will give a mean period for sixteen returns (1798–1905) of 6·685 years.

The arc of true anomaly between ☌ and P.P. being thirty days, the cometary material providing the meteoric showers must be in perihelion a month later than the observed displays.

If the periodic time of 6·68 years is correct the meteoric shower of last November took place about fifteen months before the comet reaches perihelion. The Earth is likely to intersect a part of the meteoric orbit much nearer the cometary nucleus of the swarm on November 17–21 next, and in view of recent developments the display will be awaited with great interest.

Planetary perturbation amply accounts for the removal of the shower to an earlier date than formerly. Bredichin pointed out that *Jupiter's* disturbing action in 1889·5–1891·5 brought about a reduction of 4° in the node, making the date November 23 instead of November 27, as in 1872 and 1885 ; and Dr.

Schulhof announced that there would occur another serious disturbance in 1901·2 from the same cause. Professor Abelmann, of Vienna, investigated the subject (*Ast. Nach.* 3516), and found that the node would be further affected to the extent of −6°·2, altering the shower-date to November 17 at the next maximum return in 1905 (*Observatory*, vol. xxi. p. 399). It appears that in 1901 March *Jupiter* approached the main group of *Andromedids* to within 0·5 of the Earth's mean distance from the Sun.

This particular system, from the shortness of its period, its liability to perturbation, and the great physical changes apparently affecting it, promises to give us a far clearer insight into the phenomena of meteoric streams and their cometary derivations than any other with which we are acquainted. The other prominent showers, correlated with known comets, and forming the *Lyrids*, *Perseids*, and *Leonids*, are probably of more ancient date and certainly of much longer period than the *Andromedids*. Many ages ago the former groups passed through the various gradations resolving them into annual showers with periodical maxima. No doubt, as a comet visible to human eyes, Biela's has vanished for ever. But its disintegration in 1846 and subsequent apparitions in the form of meteoric displays have added much to our knowledge of the subject.

In 1872 the *Andromedids* were confined to one night, and the stream was evidently a compact and narrow one. In 1885 the meteors were visible from November 25 to November 30, and it is probable that the particles are not only spreading out laterally, owing to repeated intersections by the earth, but that the group is lengthening out from year to year along the orbit, and will finally present an annual shower like the *Perseids*. Between the returns of 1892 and 1899 there were seven years, and between those of 1899 and 1904 only five years, so the dispersion of the meteors is evidently considerable.

Professor H. A. Newton pointed out as long ago as 1874 from a comparison of the positions of Biela's Comet at the times of the great showers of 1798, 1838, and 1872 that a "long, extended group of meteoric particles must accompany the comet in its periodic revolution, preceding it to a distance of 300 millions of miles (as in 1838) and following it to a distance of 200 millions of miles (as in 1872)" (*British Association Report*, 1875, p. 224). A further extension has probably taken place in more recent years, but the precise character of the changes effected remains to be determined by future observation.

An interesting point is that the *Andromedids* are now nearly contemporaneous with the *Leonids*. It appears likely that in ensuing years it will be possible to witness two notable meteoric displays in simultaneous action, one yielding objects having the greatest, the other the least, apparent velocity, while the green streaks of the rapid *Leonids* will contrast in a striking manner with the yellow trains of the slowly falling *Andromedids*.

Bishopston, Bristol:
1905 *May* 12.

The most Probable Position of a Point determined from the Intersections of Three Straight Lines. By S. A. Saunder, M.A.

In the course of my work on the Moon I have frequently fixed the position of what may be termed a point of the second order by measuring its position angles from three points whose coordinates had been well determined. I have shown (*B.A.A. Memoirs*, vol. vii. pp. 61–65) that when the points are near the centre of the Moon's disc, and the distances between them are small, the errors involved by neglecting the effects of libration are also small, whilst the reduction of the measures is thereby much simplified.

In the course of these reductions I have had to consider what was the most probable position of a point so determined. I am not aware what is the practice of those who compute meteoric radiants, but a recent writer (*Monthly Notices*, vol. lxv. p. 238) has assumed that the radiant should be placed at the incentre of the triangle formed by these lines. As this can be correct only under very special circumstances, I have thought it might be worth while to call attention to the point.

Supposing the determinations of the lines to be of equal weight, the conditional equations should be so stated that equal residuals are equally probable, and this would seem to be effected, at all events in my work, by writing the equations in the form $x \cos \alpha + y \sin \alpha - p = 0$, so that the residuals represent the perpendiculars from the point finally determined upon the three straight lines. If a distant point were observed with a theodolite the equations would require to be differently weighted.

If we use trilinear coordinates, taking these straight lines as sides of the triangle of reference, these three residuals will be the coordinates α, β, γ of the point, and we have to determine their values so that $\alpha^2+\beta^2+\gamma^2$ may be a minimum.

A necessary condition is that

$$a\delta\alpha + \beta\delta\beta + \gamma\delta\gamma = 0 \quad \ldots \quad \ldots \quad \ldots \quad (1)$$

and

$$a\alpha + b\beta + c\gamma = 2\Delta \quad \ldots \quad \ldots \quad \ldots \quad (2)$$

$\therefore$

$$a\delta\alpha + b\delta\beta + c\delta\gamma = 0 \quad \ldots \quad \ldots \quad \ldots \quad (3)$$

From (1) and (3) $\quad (c\alpha - a\gamma)\delta\alpha + (c\beta - b\gamma)\delta\beta = 0$

and as α, β are now independent this gives

$$\frac{a}{a} = \frac{\beta}{b} = \frac{\gamma}{c} = \frac{2\Delta}{a^2+b^2+c^2} \quad \text{by (2)}$$

If $u \equiv \alpha^2 + \beta^2 + \gamma^2$ it is easily shown by actual differentiation that

$$r = \frac{\partial^2 u}{\partial \alpha^2} = 2 + 2\frac{a^2}{c^2}, \quad s = \frac{\partial^2 u}{\partial \alpha \partial \beta} = 2\frac{ab}{c^2}, \quad t = \frac{\partial^2 u}{\partial \beta^2} = 2 + 2\frac{b^2}{c^2}$$

and hence that r, t and $rt-s^2$ are necessarily positive, which complete the conditions for a minimum.

The point thus found is the symmedian point which coincides with the incentre only when the triangle is equilateral. It may be constructed graphically from any one of the following properties :

If A B C is the triangle, I the incentre, G the centroid, K the symmedian point, AK, BK, CK, cut the sides in D, E, F respectively, and bc is an antiparallel to BC, so that $A\hat{b}c = A\hat{B}C$.

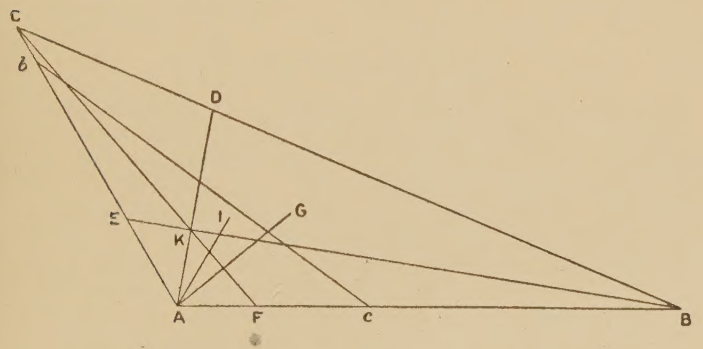
Then $$BD : DC :: c^2 : b^2$$

AK bisects bc

and $$\hat{I}AK = I\hat{A}G$$

with symmetrical relations for the other sides and angles.

This property of the symmedian point is not new, and the only object of this note is to call attention to its bearing upon the problem proposed.



If the determinations of the three lines are not of equal weights, let us suppose their weights to be l, m, n respectively. Then the condition required is that $la^2+m\beta^2+n\gamma^2$ should be a minimum ; whence, as before :

$$\frac{l\alpha}{a}=\frac{m\beta}{b}=\frac{n\gamma}{c}=\frac{2lmn\Delta}{mna^2+nlb^2+mlc^2}$$

And in the figure

$$\frac{BD}{DC}=\frac{mc^2}{nb^2},\quad \frac{CE}{EA}=\frac{na^2}{lc^2},\quad \frac{AF}{FB}=\frac{lb^2}{ma^2}$$

If the points c, b are so taken that $\frac{Ac}{Ab}=\frac{n\,.\,b}{m\,.\,c}$, bc is still bisected by AK.

Mr. H. C. Plummer has also pointed out to me that if G be

the centre of gravity of weights l, m, n at A, B, C the angles IAK, IAG are still equal, thus completing the generalisation of the properties of the symmedian point.

The weights to be given depend, in my work, only on the numbers of times the directions have been measured. In the case of meteoric paths they would seem to depend upon the length of the observed path, its nearness to recognised stars, its distance from the radiant, the persistence of a trail, and possibly upon other causes. K will coincide with I when the weights l, m, n are proportional to a, b, c.

The Cartesian coordinates of K are easily written down if we remember that a, b, c are proportional to the sines of the angles between the lines; but the expressions are so long that in practice I prefer to effect each solution by least squares.